Some new ideas to understand the dark energy

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I. General considerations

The aim of this online published article is to present a possible theoretical link between a string theory and the concept of dark energy. Our basic hypothesis is that everyone is ready to accept as a matter of facts and of observations: (a) the predominant presence of empty volumes in the universe, (b) the existence of more or less dense and organized concentrations of matter, (c) the existence of gravitational forces and, since 1998 –based on measurements concerning supernovae of type Ia-, (d) the accelerated expansion of the universe [1]. We argue that such ingredients are sufficient tools to predict an "equation-of-state (e-o-s)" for the empty volumes. We build the demonstration on very general considerations and more especially on three fundamental laws.

A. First law: Linear tensions.

Inside these empty volumes, let us consider an infinitesimal vacuum-made cylindrical piece of string with length L and section S. With the purpose to avoid a biased discussion relating two controversial items (the large amount of string theories and of models explaining dark energy [2], [3]), we shall adopt a very basic and classical behavior in doing physics. That means that we shall implicitly consider that piece of string as if it would behave classically. At least because of (d) each extremity of the string "supports" a linear tension **T** of which the main action is to elongate the string.

In fact, even if we would not know the effective nature of the force(s) which is (are) giving rise to that linear tension, the following proposition could be made:

$$\mathbf{T} = S.\frac{\partial \boldsymbol{F}}{\partial \tau}$$

Here, $\frac{\partial \boldsymbol{F}}{\partial \tau}$ is the volumetric density of force(s) acting on the extremity of the string. This is our first fundamental law.

B. Second law: Variations of the linear tensions along the time.

Otherwise it is always possible to write:

Linear tension per unit of time
=
Volumetric density of force x Surface per unit of time
=
(Volumetric density of force x length) x (length per unit of time)
=
Work of the volumetric density of force x speed
=
$\frac{Force}{volume}$ x length x speed
=
$\frac{Force}{surface}$ x speed
=
Pressure x speed

Our second fundamental law is intuitive and based on the above considerations concerning the units involved in it:

$$\frac{d\boldsymbol{T}}{dt} = p.\,\mathbf{v}$$

Here p denotes the pressure and **v** the speed at the extremity of the string.

C. Third law: pressures.

Let us now attentively consider the argumentation yielding the second law and state that it also contains the following equivalence:

Work of the volumetric density of force = pressure

This can be approximately written as follows and this will be our third law:

$$\delta p = \frac{\partial \boldsymbol{F}}{\partial \tau}\,.\,\delta\mathbf{r}$$

II. Demonstration

Let us consider the above second law and calculate an ordinary derivation along the time:

$$\frac{d^2\boldsymbol{T}}{d^2t} = \frac{dp}{dt}.\,\mathbf{v} + \mathrm{p}.\,\Gamma$$

Let us consider the third law and also calculate an ordinary derivation along the time:

$$\frac{dp}{dt} = \frac{\partial \boldsymbol{F}}{\partial\tau}\,.\,\mathbf{v} + \frac{d\frac{\partial \boldsymbol{F}}{\partial\tau}}{dt}.\,\mathbf{r}$$

Seeking for simplicity and for a first approach only we shall reduce our self to constant volumetric density of forces acting on strings:

$$\mathbf{c}^{\mathbf{te}} = \frac{\partial \boldsymbol{F}}{\partial\tau}$$

In that condition the second part of the r.h.t. giving the variations of the pressure vanishes:

$$\frac{dp}{dt} = \frac{\partial \boldsymbol{F}}{\partial\tau}\,.\,\mathbf{v}$$

And:

$$\frac{d^2\boldsymbol{T}}{d^2t} = \frac{dp}{dt}.\,\mathbf{v} + \mathrm{p}.\,\Gamma = \left(\frac{\partial \boldsymbol{F}}{\partial\tau}\,.\,\mathbf{v}\right).\,\mathbf{v} + \mathrm{p}.\,\Gamma$$

Taking into account the fact that the geometry is asymptotically Minkowskian in most parts of the universe, let us suppose that Euclidian rules for the 3D spatial part of that universe apply at the extremity of the string:

$$\forall\, \mathbf{a}, \mathbf{b}, \mathbf{c} \in E_3(K),\ \mathbf{a} \wedge (\mathbf{b} \wedge \mathbf{c}) = (\mathbf{a}.\,\mathbf{c}).\,\mathbf{b} - (\mathbf{a}.\,\mathbf{b}).\,\mathbf{c}$$

If **a = b**, then:

$$\mathbf{a} \wedge (\mathbf{a} \wedge \mathbf{c}) = (\mathbf{a}.\,\mathbf{c}).\,\mathbf{a} - (\mathbf{a}.\,\mathbf{a}).\,\mathbf{c} = (\mathbf{a}.\,\mathbf{c}).\,\mathbf{a} - a^2.\,\mathbf{c}$$

A direct application of these relations yields:

$$\frac{d^2\boldsymbol{T}}{d^2t} = \frac{dp}{dt}.\,\mathbf{v} + \mathrm{p}.\,\Gamma = \mathbf{v} \wedge \left(\mathbf{v} \wedge \frac{\partial \boldsymbol{F}}{\partial\tau}\right) + v^2.\,\frac{\partial \boldsymbol{F}}{\partial\tau} + \mathrm{p}.\,\Gamma$$

Provided the speed of the extremity of the string is not zero (which is probably the case), there is no obstruction for a rewriting such that:

$$-\frac{p}{v^2}\,.\,\Gamma + \frac{1}{v^2}.\,\frac{d^2\boldsymbol{T}}{d^2t} = \frac{\partial \boldsymbol{F}}{\partial\tau}$$

This can be seen as a special formulation of the Newton's law. For evident reasons the factor in front of the acceleration vector must be a volumetric density of matter. We can thus guess that the following equation-of-state should hold for the extremity of the string:

$$\rho = -\frac{p}{v^2}$$

This is nothing else but the e-o-s for a scenario involving a perfect fluid as explanation for the origin of dark energy in universe [2], [3].

III. Bibliography

[1] A.G. Riess et al. [Supernova Search Team Collaboration], Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, Astron. J. 116. 1009 (2008) [arXiv: astro-ph/9805201]; S. Perelmutter et al. [Supernova Cosmology Project Collaboration], Measurements of Ω and Λ from 42 High-redshift Supernovae, Astro-phys. J. 517, 565 (1999) [arXiv: astro-phy/9812133].

[2] Alternative Dark Energy Models: An Overview: J.A.S. Lima; arXiv: astro-ph/0402109v1, 4 February 2004.

[3] Comparison of dark energy models: A perspective from the latest observational data: Miao Li, Xiao-Dong Li, Xin Zhang; arXiv: 0912.3988v1, [astro-ph.CO] 20 December 2009.