Intrinsic quantization in the (E) question

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1. (E) approach of the Lorentz equation of motion in a curved space time:

1.1. Generality:

Consider, in a curved space time (I mean which could be – now and then – curved) referred to a frame with the origin O, an event (which could be placed in O at the beginning of the chronology). A force **F**(M) acts on M and therefore creates a motion with the relative speed d**OM**/ds. Per definition (see other parts of this theory to understand why we introduce this operation on a 4-dimensional vector space E [f01]), the elastic wedge product (ewp) d**OM**/ds ⌂ **F**(M) is equivalent to (coordinates language) $A_{\alpha}{}^{\gamma}{}_{\beta}.\ u^{\alpha}.\ \mathbf{F}(M)^{\beta}.\ \mathbf{e}_{\gamma}$ (1.0.0) and the ewp **F**(M) ⌂ d**OM**/ds is equivalent to $A_{\alpha}{}^{\gamma}{}_{\beta}.\ \mathbf{F}(M)^{\alpha}.\ u^{\beta}.\ \mathbf{e}_{\gamma}$ (1.0.1) [also in the coordinate's language on the same basis $\Omega(\ldots,\mathbf{e}_{\gamma},\ \ldots)$]. It follows that we can start an investigation to discover the different "kits" ([S], **s**) in which the ewp d**OM**/ds ⌂ **F**(M) can split and similarly the different kits ([P], **z**) in which the ewp **F**(M) ⌂ d**OM**/ds can split, according to the naïve intuition that each kit is (or could be) describing a "state" of the event M. But as a matter of evidence, one can suppose the existence of a great number of possible states for each given elastic wedge product at each given instant. It would be a challenge of this theory to be able to relate each split to a real situation and after that to precise with which probability each kit could appear:

d**OM**/ds ⌂ **F**(M) = $_{\phi}$[S]. **F**(M) + $_{\phi}$**s** for $\phi = 0,1,2,\ldots N$ (1.1)

If the (inner) scalar product of two vectors is also defined in the frame of the present discussion, one always can form and calculate what we called the equivalent scalar ĉ – it means in fact a scalar associated with each given kit ([S], **s**) and defined by:

F(M). {[S]. **F**(M) **-** d**OM**/ds ⌂ **F**(M) + **s**} – ĉ = 0 (1.2)

It is true that we could associate an infinity number of scalars with each kit, each of these scalars depending of the vector and of the position of this peculiar vector (on the left or on the right) implied in the scalar product with {[S]. **F**(M) **-** d**OM**/ds ⌂ **F**(M) + **s**}. In a first approach taking its source in a trivial comparison between the wedge product and the ewp, it is perhaps relevant to reduce the number of these scalars to those resulting of the scalar product with

F(M) or with d**OM**/ds. We usually consider only those resulting of the scalar product with the second vector involved in a given ewp that means here with **F**(M).
The standardized expression of this scalar is (see other part of this theory):
$\sum_\gamma \sum_\delta [\sum_\varepsilon g_{\gamma\varepsilon}. p_{\varepsilon\delta} - (\sum_\eta \sum_\varepsilon A_\eta{}^\varepsilon{}_\delta. u^\eta. g_{\gamma\varepsilon})]. w^\gamma. w^\delta + \sum_\gamma \sum_\delta g_{\gamma\delta}. w^\gamma. z^\delta - \hat{c} = 0$ (1.3)
One only needs to write :
[P] = [S] (1.4)
w = **F**(M) (1.5)
u = d**OM**/ds (1.6)
z = **s** (1.7)
to get the valid expression of only one "equivalent" scalar for this section:
$\sum_\gamma \sum_\delta [\sum_\varepsilon g_{\gamma\varepsilon}. s_{\varepsilon\delta} - (\sum_\eta A_\eta{}^\varepsilon{}_\delta. (d\mathbf{OM}/ds)^\eta. g_{\gamma\varepsilon})]. (\mathbf{F}(M))^\gamma. (\mathbf{F}(M)^\delta + \sum_\gamma \sum_\delta g_{\gamma\delta}. (\mathbf{F}(M))^\gamma. s^\delta - \hat{c} = 0$
(1.8)

The GR approach postulates:
F(M) = m. D**u**/ds [17; page 106; (20.3)]
In the coordinate's language :
$F(M)^\gamma = m. Du^\gamma/ds \ \forall\ \gamma = 0,1,2,3.$ (1.9)
According to [21 ; pages 95-96 ; § 34] :
$Du^\gamma/ds = \sum_\lambda D_\lambda u^\gamma. dx^\lambda/ds$ (1.10)
It follows :
$F(M)^\gamma = m. \sum_\lambda D_\lambda u^\gamma. dx^\lambda/ds$ (1.11)
and :
$\mathbf{F}(M) = m. T_2 (°)(\mathbf{D_x}, \mathbf{u}). \mathbf{u}$ (1.12)
An "equivalent" scalar:
$\sum_\gamma \sum_\delta [\sum_\varepsilon g_{\gamma\varepsilon}. s_{\varepsilon\delta} - (\sum_\eta A_\eta{}^\varepsilon{}_\delta. (d\mathbf{OM}/ds)^\eta. g_{\gamma\varepsilon})]. (\mathbf{F}(M))^\gamma. (\mathbf{F}(M)^\delta + \sum_\gamma \sum_\delta g_{\gamma\delta}. (\mathbf{F}(M))^\gamma. s^\delta - \hat{c} = 0$
becomes:
$\sum_\gamma \sum_\delta [\sum_\varepsilon g_{\gamma\varepsilon}. s_{\varepsilon\delta} - (\sum_\eta A_\eta{}^\varepsilon{}_\delta. u^\eta. g_{\gamma\varepsilon})]. (m. \sum_\lambda D_\lambda u^\gamma. u^\lambda). (m. \sum_\mu D_\mu u^\delta. u^\mu)$
$+ \sum_\gamma \sum_\delta g_{\gamma\delta}. (m. \sum_\lambda D_\lambda u^\gamma. u^\lambda). s^\delta - \hat{c} = 0$ (1.13)
and at the end:
$m^2. \sum_\lambda \sum_\mu \sum_\gamma \sum_\delta [\sum_\varepsilon g_{\gamma\varepsilon}. s_{\varepsilon\delta} - (\sum_\eta A_\eta{}^\varepsilon{}_\delta. u^\eta. g_{\gamma\varepsilon})]. D_\lambda u^\gamma. D_\mu u^\delta. u^\lambda. u^\mu$
$+ m. \sum_\lambda \sum_\gamma \sum_\delta g_{\gamma\delta}. D_\lambda u^\gamma. s^\delta. u^\lambda. - \hat{c} = 0$ (1.14)
That is in fact: for a set of possible kits (${}_\phi$[S], ${}_\phi$**s**), we have an associated set of possible scalars:
$m^2. \sum_\lambda \sum_\mu \sum_\gamma \sum_\delta [\sum_\varepsilon g_{\gamma\varepsilon}. {}_\phi s_{\varepsilon\delta} - (\sum_\eta A_\eta{}^\varepsilon{}_\delta. u^\eta. g_{\gamma\varepsilon})]. D_\lambda u^\gamma. D_\mu u^\delta. u^\lambda. u^\mu$
$+ m. \sum_\lambda \sum_\gamma \sum_\delta g_{\gamma\delta}. D_\lambda u^\gamma. {}_\phi s^\delta. u^\lambda. - {}_\phi\hat{c} = 0$ for $\phi = 0,1,2, \ldots N$ (1.14.ϕ)
The only real complication will appear later for the choice of four kits (${}_\phi$[S], ${}_\phi$**s**) under the N possible kits when we will have to compare two equivalent scalars each related to comparable ewp. This will force us first to define the term: "comparable ewp". Energetic and topological considerations certainly help to get the correct answers.

1.2. Commentaries 01 :

The relation (1.14) implicitly contains the ability to describe a large number of situations ϕ for a mass m and suggests that all of them can be understood as a peculiar configuration resuming in ${}_\phi$[S], ${}_\phi$**s**, ${}_\phi\hat{c}$ of a sophisticated interaction between the local geometry [defined by the fundamental tensor and its representative matrix $T_2(.)(\Omega, \Omega)$ - and whose instantaneous evolution is given by the Christoffel's symbols- (not directly appearing here)], the local topology [in someway related to the coefficients $A_\eta{}^\varepsilon{}_\delta$ defining the ewp on Ω] and the flow **u**. This relation (1.14) is a conoide relative to the coordinates of **u** and a polynomial form of degree 2 relative to the mass m. So it is also amazingly containing a strange possibility, in extenso: some cinematic and topological circumstances could be associated with "twin - particles". Leads this opportunity to the EPR paradox…? Before going so far, just consider a very simple experiment that you can do in your bathroom. Fill your bath with enough water; wait a while, until the water does no more move; put your hand slowly into the water and now push it strongly in a direction. If you are lucky, you could see two whirls going in the direction where your hand was pushing and each of it turning in an opposite sense than the other. Similar things appear in swimming pools with under-water streams. Two photons hurting each other with enough energy generate one electron and one positron.

1.3. The usual Lorentz equation of motion in curved space time:
Observing the usual formulation of these equations [16; page 68; (33-1)] where k is (q / m), q the charge and m the mass of the particle :
$d^2x^\theta/ds^2 + \sum_\gamma \sum_\delta \Gamma_\gamma{}^\theta{}_\delta . (dx^\gamma/ds). (dx^\delta/ds) = k. \sum_\gamma F^\theta{}_\gamma . (dx^\gamma/ds)$ [16 ; page 68 ; (33-1)]
reorganizing these four equations as follows :
$\sum_\gamma \sum_\delta \Gamma_\gamma{}^\theta{}_\delta . (dx^\gamma/ds). (dx^\delta/ds) - k. \sum_\gamma F^\theta{}_\gamma . (dx^\gamma/ds) + d^2x^\theta/ds^2 = 0$; $\theta = 0,1,2,3.$
One easily recognizes four « conoides » concerning the four contra-variant coordinate's u^λ of **u** :
$\sum_\gamma \sum_\delta \Gamma_\gamma{}^\theta{}_\delta . u^\gamma. u^\delta - k. \sum_\gamma F^\theta{}_\gamma . u^\gamma + du^\theta/ds = 0$; $\theta = 0,1,2,3.$ (1.23)

Each of these four conoides could be *a priori* interpreted as the equivalent scalar (see generality above) for a family of splitting of a set of elastic wedge product $_\phi\mathbf{y} \wedge d\mathbf{OM}/ds$ where the vectors $_\phi\mathbf{y}$ are actually unknown and $\phi = 0, 1, \ldots, N$. Observing carefully (1.23) we understand the necessity to consider splitting of the following formalism ($_\phi$[P], $_\theta\mathbf{z}$) where θ = 0, 1, 2 and 3 only because the linear part $\sum_\mu g_{\lambda\mu}.. z^\mu$ depends of the θ coordinate. Effectively, writing :
$_\phi\mathbf{y} \wedge d\mathbf{OM}/ds = {}_\phi[P]. d\mathbf{OM}/ds + {}_\theta\mathbf{z}$ (1.24)
If (as we suppose in this section of the theory) the inner scalar product of two vectors is also defined in the frame of the present discussion, one always can form and calculate what we called the equivalent scalar:
$d\mathbf{OM}/ds. \{_\phi\mathbf{y} \wedge d\mathbf{OM}/ds - {}_\phi[P]. d\mathbf{OM}/ds + {}_\theta\mathbf{z}\} - {}_\theta S = 0$ (1.25)
this is, in the coordinate's language:
$\sum_\gamma \sum_\delta [\sum_\varepsilon g_{\gamma\varepsilon}. {}_\phi p_{\varepsilon\delta} - (\sum_\eta \sum_\varepsilon A_\eta{}^\varepsilon{}_\delta. {}_\phi y^\eta. g_{\gamma\varepsilon})]. u^\gamma. u^\delta + \sum_\gamma \sum_\delta g_{\gamma\delta}. u^\gamma. {}_\theta z^\delta - {}_\theta S = 0$ (1.26)
Observing carefully (1.23) again we also understand the necessity to relate the ($_\phi$[P], $_\phi\mathbf{y}$) to the θ coordinate and this is automatically leading to:
$\sum_\gamma \sum_\delta [\sum_\varepsilon g_{\gamma\varepsilon}. {}_\theta p_{\varepsilon\delta} - (\sum_\eta \sum_\varepsilon A_\eta{}^\varepsilon{}_\delta. {}_\theta y^\eta. g_{\gamma\varepsilon})]. u^\gamma. u^\delta + \sum_\gamma \sum_\delta g_{\gamma\delta}. u^\gamma. {}_\theta z^\delta - {}_\theta S = 0$ (1.27.θ)
making a comparison with (1.23) possible if we write:
$\Gamma_\gamma{}^\theta{}_\delta = [\sum_\varepsilon g_{\gamma\varepsilon}. {}_\theta p_{\varepsilon\delta} - (\sum_\eta \sum_\varepsilon A_\eta{}^\varepsilon{}_\delta. {}_\theta y^\eta. g_{\gamma\varepsilon})]$; $\forall\gamma\ \forall\delta\ \forall\theta = 0,1,2,3.$ (1.28.θ)
$- k. F^\theta{}_\gamma = \sum_\delta g_{\gamma\delta}. {}_\theta z^\delta$; $\forall\gamma\ \forall\theta = 0,1,2,3.$ (1.29.θ)
$du^\theta/ds = - {}_\theta S$; $\forall\theta = 0,1,2,3.$ (1.30.θ)

commentaries 02 :
The theory (E) is *obliged* to associate *four kits* ($_\theta$[P], $_\theta\mathbf{z}$) with *one* real particle; the contra-variant coordinates of the four vector-components of these four kits always can be organized inside a matrix and because of this we can conclude that this theory associates five matrix to one real particle. It is certainly too soon but not forbidden to mention the Pauli's matrixes. This is only an apparently complicated situation because all elements of this new description receive an interpretation that we will now evocate briefly.
1°) The four matrixes $_\theta$[P] are always connected to four other matrixes with which we can build what we call the cube of Christoffel ▼Γ of the basis Ω in which the discussion takes place. For a given basis Ω with a given topology (that means here ▼Γ, ▼A and T_2(.)(Ω, Ω) are given) these four matrixes $_\theta$[P] only depends of the $_\theta\mathbf{y}$. In any flat space (▼Γ = ▼0) preserving the 3-dimensional volumes *[1] the relations (1.28.θ ; ∀θ = 0, 1, 2, 3) are reduced in:

$0 = [\sum_\varepsilon g_{\gamma\varepsilon} .\ {}_\theta p_{\varepsilon\delta} - (\sum_\eta {}_\theta y^\eta .\ e_{\gamma\eta\delta})]$; $\forall\gamma\ \forall\delta$

Other remark: if the matrix components of the four kits associated with one particle are simultaneously the most trivial, this particle is moving in an inertial frame (e.g. a Lorentz's basis); that is, if:
$\forall\ \theta = 0,1,2,3$; $({}_\theta[P], {}_\theta\mathbf{z}) = (\Phi(\mathbf{y}), {}_\theta\mathbf{z})$,
then :
$\forall\ \theta = 0,1,2,3$;
$\Gamma_\gamma{}^\theta{}_\delta$
$= [\sum_\varepsilon g_{\gamma\varepsilon} .\ {}_\theta p_{\varepsilon\delta} - (\sum_\eta \sum_\varepsilon A_\eta{}^\varepsilon{}_\delta .\ y^\eta .\ g_{\gamma\varepsilon})]$
$= [\sum_\varepsilon g_{\gamma\varepsilon} .\ (\sum_\eta A_\eta{}^\varepsilon{}_\delta .\ y^\eta) - (\sum_\eta \sum_\varepsilon A_\eta{}^\varepsilon{}_\delta .\ y^\eta .\ g_{\gamma\varepsilon})]$
$= 0$

2°) The four vectors ${}_\theta\mathbf{z}$ are directly connected to the matrix representation of the EM field strength-tensor in the basis Ω because one can always write (1.29.θ) :
$- k.\ F^\theta{}_\lambda = - k.\ g_{\lambda\mu} .\ F^{\theta\mu} = \sum_\mu g_{\lambda\mu} ..\ {}_\theta z^\mu$
leading to :
$- k.\ F^{\theta\mu} = {}_\theta z^\mu$
The EM field strength-tensor in a given basis Ω of space time E determinates the four vector components of the four kits associated with any given particle.

3°) The four scalars ${}_\theta S$ clearly represent the components of the relative acceleration of the particle in the basis Ω :
$d\mathbf{u}/ds + \mathbf{S} = \mathbf{0}$ (1.30)

1.4. The connection:
A door has been opened by the preceding description and concerns each vector ${}_\theta\mathbf{y}$ which is totally indeterminate (or determinate by circumstances that the particle encounters) and which influence is limited to the matrix components ${}_\theta[P]$ through the relation (1.28.θ). Apparently, nothing can forbid that we investigate what would happen if ${}_\theta\mathbf{y} = \mathbf{y} = \mathbf{F}(M)$ (1.31). In doing so we would be making a peculiar mathematical application of the method explained above § 1.3. and get four splitting ${}_\theta[P].\ d\mathbf{OM}/ds + {}_\theta\mathbf{z}$ of the elastic wedge product **F**(M) ⌂ d**OM**/ds:
F(M) ⌂ d**OM**/ds = ${}_\theta[P].\ d\mathbf{OM}/ds + {}_\theta\mathbf{z}$ for $\theta = 0,1,2,3$.
This introduces the ticklish question of a comparison with the elastic wedge product d**OM**/ds ⌂ **F**(M). These two products are generally different and define two families of equivalent scalars. The ewp **F**(M) ⌂ d**OM**/ds generates the d**OM**/ds. {**F**(M) ⌂ d**OM**/ds - ${}_\theta[P].\ d\mathbf{OM}/ds + {}_\theta\mathbf{z}\} - {}_\theta S = 0$ for $\theta = 0, 1, 2$ and 3 family, called F_2, whilst the ewp d**OM**/ds ⌂ **F**(M) generates the $m^2.\ \sum_\lambda \sum_\mu \sum_\gamma \sum_\delta [\sum_\varepsilon g_{\gamma\varepsilon} .\ {}_\phi s_{\varepsilon\delta} - (\sum_\eta A_\eta{}^\varepsilon{}_\delta .\ u^\eta .\ g_{\gamma\varepsilon})].\ D_\lambda u^\gamma .\ D_\mu u^\delta .\ u^\lambda .\ u^\mu + m.\ \sum_\lambda \sum_\gamma \sum_\delta g_{\gamma\delta} .\ D_\lambda u^\gamma .\ {}_\phi s^\delta .\ u^\lambda .\ - {}_\phi\hat{c} = 0$ for $\phi = 0, 1, 2, \ldots$ N family, called F_1. Both families, F_1 and F_2, are sets of conoides concerning the four contra-variant coordinates u^λ of **u**.
Until now, we did not precise what kind of force and what force exactly was acting. The force involved in F_1 corresponds to any force. It could be any force acting in space time and eventually, why not, either this random force resulting from the random fluctuations of the ZPF or the Lorentz force, or a combination of several types of forces. Any set of four members of this family F_1 can always be mathematically compared with the four members of the F_2 family. But we unfortunately encounter an important problem with the definition of the force (1.31) generating the F_2 family because this family itself is resulting of [16; page 68; (33-1)] which is equating the general relativistic representation of the force with a particular one, i.e. the Lorentz force. That is the scalars of this family F_2 are already related to a particle immerged in an EM field and, per construction, undergoing the influence of a set of four

vectors (…, ${}_{\theta}\mathbf{y}$, …) which is exceptionally here the set (**F**(M), **F**(M), **F**(M), **F**(M)) where **F**(M) is a priori any other force. That is, it is difficult to give a sense to a comparison with elements of the first family, F_1.

Nevertheless any particle immerged in an EM field automatically satisfies a set of four elements of F_2 – per construction – which could be the image of four elements of F_1. To get this, we just have to imagine a mathematical function $\Im$: $\mid$ $(f_0, f_1, f_2, f_3) \in F_1 \times F_1 \times F_1 \times F_1 \rightarrow \Im(f_0, f_1, f_2, f_3) = (g_0, g_1, g_2, g_3) \in F_2 \times F_2 \times F_2 \times F_2$. It could make sense if we suppose that generalized Lorentz equations of motion for any particle proposed *in this theory*, in extenso the element (g_0, g_1, g_2, g_3) of $F_2 \times F_2 \times F_2 \times F_2$, are a peculiar example of a generalized formulation of any motion defined by an element (f_0, f_1, f_2, f_3) of $F_1 \times F_1 \times F_1 \times F_1$. This special example automatically including the influence of an EM field and eventually accepting the external influence of **F**(M).

A consequence of this way of thinking, if true, is that the external influence, i. e. here the force **F**(M), could be reduced to the Lorentz force $\mathbf{F}_{EM}$(M) itself; this would mean consequently that generalized Lorentz equations of motion proposed in this theory would describe the interaction of a charged particle immerged in an EM field with itself when (…, ${}_{\theta}\mathbf{y}$, …) = ($\mathbf{F}_{EM}$(M), $\mathbf{F}_{EM}$(M), $\mathbf{F}_{EM}$(M), $\mathbf{F}_{EM}$(M)); what is usually called the self-interaction and related to the two-body problem (A trajectory that crosses it own light cone, emanating from any 4-point on the trajectory, cannot be regard as a single charged particle [26]).

An other consequence of this way of thinking, if true, is that the external influence could be any force **F**(M), not necessary reduced to the Lorentz force $\mathbf{F}_{EM}$(M) and this would mean that generalized Lorentz equations of motion proposed in this theory would describe the interaction of a charged particle immerged in an EM field with any external field of force including automatically as a matter of fact its own field of forces [the self-interaction part obligatory contained in F(M)] when (…, ${}_{\theta}\mathbf{y}$, …) = (**F**(M), **F**(M), **F**(M), **F**(M)). From this instant forward we accept this way of thinking and test it. The result is that these two products have always complicated connections given by a system (1.36) that we will demonstrate now.

Before doing it, we recall two of many reasons why we introduced this elastic wedge product which is sometimes so akin with the usual wedge product:

i) the possibility to incorporate a deformation of the geometry as it certainly appears in real circumstances, e. g. distance dilatation and time contraction due to a high speed (in the restricted relativity) and deformation of space time because of the presence of mass (in the general relativity);

ii) the possibility to incorporate the description of central forces **F**(M) = (scalar). **OM**, and consequently of following elastic wedge products (scalar). **OM** ⌂ d**OM**/ds which are - per construction – related to the usual well-known quantified **OM** ∧ d**OM**/ds products. All these ingredients (EM field, central gravitational and electric fields, quantified kinetic momentum, electron surrounding the proton at about c-speed) are present, for example (but the remark is valid for any other atom), in an atom of hydrogen which – in despite of this exotic mixture between general relativity and quantum theory – owns stabile orbital.

Our way of thinking leads to a comparison between:

$$m^2. \sum_\lambda \sum_\mu \sum_\gamma \sum_\delta [\sum_\varepsilon g_{\gamma\varepsilon}.\, {}_{\theta}s_{\varepsilon\delta} - (\sum_\eta A_\eta{}^{\varepsilon}{}_{\delta}.\, u^\eta.\, g_{\gamma\varepsilon})].\, D_\lambda u^\gamma.\, D_\mu u^\delta.\, u^\lambda.\, u^\mu$$
$$+ m. \sum_\lambda \sum_\gamma \sum_\delta g_{\gamma\delta}.\, D_\lambda u^\gamma.\, {}_{\theta}s^\delta.\, u^\lambda. - {}_{\theta}\hat{c} = 0 \qquad (1.14.\theta)$$

and (1.27.θ) when one replace **y** by **F**(M):

$$\sum_\lambda \sum_\mu [\sum_\varepsilon g_{\lambda\varepsilon}.\, {}_{\theta}p_{\varepsilon\mu} - (\sum_\eta \sum_\varepsilon A_\eta{}^{\varepsilon}{}_{\mu}.\, (\mathbf{F}(M))^\eta.\, g_{\lambda\varepsilon})].\, u^\lambda.\, u^\mu + \sum_\lambda \sum_\mu g_{\lambda\mu}.\, u^\lambda.\, {}_{\theta}z^\mu - {}_{\theta}S = 0 \qquad (1.32.\theta)$$

From this comparison it follows the equations :

$m^2. \sum_\gamma \sum_\delta [\sum_\varepsilon g_{\gamma\varepsilon}. {}_\theta s_{\varepsilon\delta} - (\sum_\eta A_\eta{}^\varepsilon{}_\delta. u^\eta. g_{\gamma\varepsilon})]. D_\lambda u^\gamma. D_\mu u^\delta$

$= \chi. [\sum_\varepsilon g_{\lambda\varepsilon}. {}_\theta p_{\varepsilon\mu} - (\sum_\eta \sum_\varepsilon A_\eta{}^\varepsilon{}_\mu. (\mathbf{F}(M))^\eta. g_{\lambda\varepsilon})]$ (1.33.θ)

$m. \sum_\gamma \sum_\delta g_{\gamma\delta}. D_\lambda u^\gamma. {}_\theta s^\delta = \chi. \sum_\mu g_{\lambda\mu}.. {}_\theta z^\mu$ (1.34.θ)

${}_\theta\hat{c} = \chi. {}_\theta S$ (1.35.θ)

Recalling (1.11), (1.28, 29, 30.θ), one gets:

$m^2. \sum_\gamma \sum_\delta \sum_\varepsilon [g_{\gamma\varepsilon}. {}_\theta s_{\varepsilon\delta} - (\sum_\eta A_\eta{}^\varepsilon{}_\delta. u^\eta. g_{\gamma\varepsilon})]. D_\lambda u^\gamma. D_\mu u^\delta$

$= \chi. \sum_\varepsilon [g_{\lambda\varepsilon}. {}_\theta p_{\varepsilon\mu} - m. \sum_\eta \sum_\pi A_\eta{}^\varepsilon{}_\mu. D_\pi u^\eta. u^\pi. g_{\lambda\varepsilon}]$ (1.36)

$m. (\mathbf{D}_\lambda \mathbf{u}. {}_\theta\mathbf{s}) = - k. \chi. F^\theta{}_\lambda$ (1.36.2)

$\chi. d\mathbf{u}/ds + \hat{\mathbf{c}} = \mathbf{0}$ (1.36.1)

1.5. Commentaries 03; introducing the problem of the mass:

These equations implicitly define the domain of validity of this approach. The problem is to recognize this domain. We make here short commentaries about the system (1.36) and introduce the discussion concerning the mass.

The result (1.36.1) demonstrates that the unidentified vector $\hat{\mathbf{c}}$ is proportional to the relative acceleration of the particle.

The second one (1.36.2) tells a similar interrogation than the Lorentz equations itself because is strongly suggesting that an EM field could generate a mass… Even if a lot of physicists are dreaming of this result (the mass has an EM origin), we have to manage carefully such too hopeful interpretation. An intuition related to our way of thinking explained above just tells that this approach may be have a connection with the "magneto-mechanical anomaly" (see Pauli equation for the "Spin-Bahn Kopplung" and all explanations about the "Zitterbewegung").

Another special and indirect consequence of (1.36.2) that we will use later is:

$m. (\mathbf{D}_\lambda \mathbf{u}. {}_\theta\mathbf{s}) = - k. \chi. \sum_\mu g_{\lambda\mu}. F^{\theta\mu}$

giving in a Lorentz basis:

$m. (\mathbf{D}_\lambda \mathbf{u}. {}_\theta\mathbf{s}) = - k. \chi. \sum_\mu \eta_{\lambda\mu}. F^{\theta\mu} = - k. \chi. \eta_{\lambda\lambda}. F^{\theta\lambda} = \pm k. \chi. F^{\theta\lambda}$

the usual structure of the EM field tensor (anti-symmetric tensor with a diagonal equal to zero) leads to:

$(\mathbf{D}_\lambda \mathbf{u}. {}_\theta\mathbf{s}) + (\mathbf{D}_\theta \mathbf{u}. {}_\lambda\mathbf{s}) = 0 ; \forall\lambda, \forall\theta.$ (1.36.2.Lorentz)

Concerning the result (1.36), we must ask exactly the same question than this one resulting of (1.14) (see commentaries 01); that is, we are obliged to discuss an embarrassing question about the possible duality of this mass m.

Before doing this, we remark that (1.36) is the sum of terms of the second and of the third degree in the different coordinates of **u** and that we could make in certain circumstances an identification term by term in braking (1.36) into two independent parts and write:

1°) $m^2. \sum_\gamma \sum_\delta \sum_\varepsilon g_{\gamma\varepsilon}. {}_\theta s_{\varepsilon\delta} = \chi. \sum_\varepsilon g_{\lambda\varepsilon}. {}_\theta p_{\varepsilon\mu} ; \forall\lambda, \forall\mu, \forall\theta$ (1.36.2)

Consider the matrix product $\hat{G}. {}_\theta[S]$ where $\hat{G} = T_2(.)(\Omega, \Omega)$ is the fundamental matrix of the basis Ω (to simplify the notation); the (γ, δ) term of this product is $\sum_\varepsilon g_{\gamma\varepsilon}. {}_\theta s_{\varepsilon\delta}$. Consider now the matrix product $\hat{G}. {}_\theta[P]$; the (λ, μ) term of this product is $\sum_\varepsilon g_{\lambda\varepsilon}. {}_\theta p_{\varepsilon\mu}$. Conclusion: χ times the (λ, μ) term of $\hat{G}. {}_\theta[P]$ is m² times the sum of all terms of the $\hat{G}. {}_\theta[S]$ matrix and consequently all terms of $\hat{G}. {}_\theta[P]$ are equal. We cannot believe that this situation represents something else than only a very peculiar one

2°) m. $\sum_\gamma \sum_\delta \sum_\varepsilon \sum_\eta A_\eta{}^\varepsilon{}_\delta.\ u^\eta.\ g_{\gamma\varepsilon}.\ D_\lambda u^\gamma.\ D_\mu u^\delta = \chi.\ \sum_\varepsilon \sum_\eta \sum_\pi A_\eta{}^\varepsilon{}_\mu.\ D_\pi u^\eta.\ u^\pi.\ g_{\lambda\varepsilon}$

m. $\sum_\gamma \sum_\delta \sum_\varepsilon \sum_\eta A_\eta{}^\varepsilon{}_\delta.\ u^\eta.\ g_{\gamma\varepsilon}.\ D_\lambda u^\gamma.\ D_\mu u^\delta = \chi.\ \sum_\varepsilon \sum_\kappa \sum_\eta A_\kappa{}^\varepsilon{}_\mu.\ D_\eta u^\kappa.\ u^\eta.\ g_{\lambda\varepsilon}$

m. $\sum_\gamma \sum_\delta \sum_\varepsilon A_\eta{}^\varepsilon{}_\delta.\ g_{\gamma\varepsilon}.\ D_\lambda u^\gamma.\ D_\mu u^\delta = \chi.\ \sum_\varepsilon \sum_\kappa A_\kappa{}^\varepsilon{}_\mu.\ D_\eta u^\kappa.\ g_{\lambda\varepsilon}\ ;\ \forall\lambda, \forall\mu, \forall\eta$ (1.36.3)

This result is quite more difficult to summarize and certainly indicates the complicated connections between the force and the geometry of space time. *[1] Remember that deformation of the space preserving the quantity $\tau = |\mathbf{u}.\ (\mathbf{v} \wedge \mathbf{w})|$ (which is a 3-dimensional volume if expressed in a "classic" space) result in:

$\sum_\delta g_{\gamma\delta}.\ A_\lambda{}^\delta{}_\mu = e_{\gamma\lambda\mu}$ (1.37)

Suppose *a priori* that (1.37) is valid and apply it to (1.36.3) :

m. $\sum_\gamma \sum_\delta \sum_\varepsilon A_\eta{}^\varepsilon{}_\delta.\ g_{\gamma\varepsilon}.\ D_\lambda u^\gamma.\ D_\mu u^\delta = \chi.\ \sum_\varepsilon \sum_\kappa A_\kappa{}^\varepsilon{}_\mu.\ D_\eta u^\kappa.\ g_{\lambda\varepsilon}\ ;\ \forall\lambda, \forall\mu, \forall\eta$

m. $\sum_\gamma \sum_\delta e_{\gamma\eta\delta}.\ D_\lambda u^\gamma.\ D_\mu u^\delta = \chi.\ \sum_\kappa e_{\lambda\kappa\mu}.\ D_\eta u^\kappa;\ \forall\lambda, \forall\mu, \forall\eta$ (1.38)

In any case, the identification neither avoids the embarrassing discussion about the duality of the mass nor represents a general case. The system (1.36.2) just suggests that each EM field could generate a mass or, better said, could have a "geometric equivalent" given by the left term of (1.36.2). Reading the Lorentz equations [16; page 68; (33-1)] from the right side to the left side we get exactly this same strange sensation. Since these two quantities (the mass and the charge of a given particle) and since the left part and the right part of these equations have in fact an independent existence, this situation really tells a problem because Lorentz equations are supposed to describe any charged or uncharged particle and are a priori not appropriate to connect the charge and the mass of any given particle. Our approach indirectly leads to modern considerations concerning some attempts proposing to explain the origin of the mass of a charged particle (e.g. the electron) by an interaction with the EM field, eventually with the EM field in its lower energetic level, i.e. the ZPF.

2. General considerations:

The method exposed above (see § 1.4) does not immediately identify the four vectors $_\theta\mathbf{s}$ or, better said, connects these vectors to the special conditions (always including at least an EM component due to the ZPF, and eventually some more EM components) encountered by the particle. In despite of this disturbing relationship between mass and charge of a particle, the (1.36.2) formalism contains implicitly the beginning of an interpretation for the $_\theta\mathbf{s}$ vectors and consequently for the components of the EM field tensor. They appear to be the different "projections" of these unidentified vectors $_\theta\mathbf{s}$ on the $\mathbf{D}_\lambda\mathbf{u}$ vectors. In fact, each special condition encountered by the particle give a face to the $\mathbf{D}_\lambda\mathbf{u}$ vectors and helps to identify precisely the $_\theta\mathbf{s}$ vectors. In the way around, the relation (1.36.2) is a very general one and many different situations related to the $\mathbf{D}_\lambda\mathbf{u}$ and to the $_\theta\mathbf{s}$ could add an EM contribution if connected in the nature with a "non zero mass and charge" particle.

2.1. The force F(M) is the Lorentz force:

2.1.1. Interpretation of the vector components of the kits:

This paragraph is analyzing what happens if $\mathbf{D}_\lambda\mathbf{u}$ is resulting of the EM field itself? That is if we suppose that all external influences on the particle immerged in the EM field are reduced to EM field itself (self-interaction):

$\mathbf{F}(M) = \partial T^{\alpha\beta}/\partial x^\beta.\ \mathbf{e}_\alpha = (q/c).\ F^{\alpha\beta}.\ u_\beta.\ \mathbf{e}_\alpha$ (1.41)

Or equivalently in the coordinates language:

$F(M)^\alpha = (q/c).\ F^{\alpha\beta}.\ u_\beta = (q/c).\ F^{\alpha\beta}.\ g_{\beta\lambda}.\ u^\lambda$ (1.42)

Comparing with the other and general formalism (1.11) of a force:

$F(M)^\alpha = m.\ \sum_\lambda D_\lambda u^\alpha.\ u^\lambda$

One gets:

$m.\ \sum_\lambda D_\lambda u^\alpha.\ u^\lambda = (q/c).\ F^{\alpha\beta}.\ g_{\beta\lambda}.\ u^\lambda$ (1.43)

And writing this independently of the flow:
$m.\ D_\lambda u^\alpha = (q/c).\ F^{\alpha\beta}.\ g_{\beta\lambda}$ (1.44)
it follows:
$\mathbf{D_\lambda u} = (D_\lambda u^0, D_\lambda u^1, D_\lambda u^2, D_\lambda u^3) = (k/c).\ (F^{0\beta}.\ g_{\beta\lambda}, F^{1\beta}.\ g_{\beta\lambda}, F^{2\beta}.\ g_{\beta\lambda}, F^{3\beta}.\ g_{\beta\lambda})$ (1.45)
If the fundamental tensor is supposed to be symmetric:
$\mathbf{D_\lambda u} = (k/c).\ (F^{0\beta}.\ g_{\lambda\beta}, F^{1\beta}.\ g_{\lambda\beta}, F^{2\beta}.\ g_{\lambda\beta}, F^{3\beta}.\ g_{\lambda\beta}) = (k/c).\ (F^0{}_\lambda, F^1{}_\lambda, F^2{}_\lambda, F^3{}_\lambda)$
$\mathbf{D_\lambda u} = (k/c).\ F^\alpha{}_\lambda.\ \mathbf{e}_\alpha$ (1.46)
Consequently, one gets the scalar product:
$(\mathbf{D_\lambda u}.\ {}_\theta\mathbf{s}) = (k/c).\ g_{\alpha\beta}.\ F^\alpha{}_\lambda.\ {}_\theta s^\beta$ (1.47)
But the relation (1.36.2) was:
$m.\ (\mathbf{D_\lambda u}.\ {}_\theta\mathbf{s}) = -\ k.\ \chi.\ F^\theta{}_\lambda$
It results:
$(k/c).\ g_{\alpha\beta}.\ F^\alpha{}_\lambda.\ {}_\theta s^\beta = -\ (k.\ \chi/m).\ F^\theta{}_\lambda$ (1.48)
leading to:
$g_{\alpha\beta}.\ F^\alpha{}_\lambda.\ {}_\theta s^\beta = -\ (\chi.\ c\ /\ m).\ F^\theta{}_\lambda$
$F^\alpha{}_\lambda.\ {}_\theta s_\alpha = -\ (\chi.\ c\ /\ m).\ F^\theta{}_\lambda$
${}_\theta s_\alpha = -\ (\chi.\ c\ /\ m).\ \delta^\alpha{}_\theta$ (1.49)
The ${}_\theta\mathbf{s}^*$ vectors are the basis vectors $\mathbf{e}_\theta$.
${}_\theta\mathbf{s}^* = -\ (\chi.\ c\ /\ m).\ \mathbf{e}_\theta$ (1.50)
This is the first important result of the splitting method exposed here.
Any basis Ω of a 4-dimensional space possessing a symmetric fundamental tensor $T_2(.)(\Omega, \Omega)$ naturally supplies a set of four vectors ${}_\phi\mathbf{s}$ to express four splitting of the ewp $d\mathbf{OM}/ds \ \triangle\ \mathbf{F}(M) = {}_\phi[S].\ \mathbf{F}(M) + {}_\phi\mathbf{s}$ when and if the only external force $\mathbf{F}(M)$ acting on a particle immerged in an EM field is the EM Lorentz force $\mathbf{F}_{EM}(M)$.

Now recall (1.34.θ):
$m.\ \Sigma_\gamma \Sigma_\delta\ g_{\gamma\delta}.\ D_\lambda u^\gamma.\ {}_\theta s^\delta = \chi.\ \Sigma_\mu\ g_{\lambda\mu}.\ {}_\theta z^\mu$
Make use of (1.49)
$-\ (\chi.\ c\ /\ m).\ m.\ \Sigma_\gamma\ D_\lambda u^\gamma.\ \delta^\gamma{}_\theta = \chi.\ \Sigma_\mu\ g_{\lambda\mu}.\ {}_\theta z^\mu$ (1.51)
$-\ c.\ D_\lambda u^\theta = \Sigma_\mu\ g_{\lambda\mu}.\ {}_\theta z^\mu$ (1.52)
Recall (1.46):
$-\ k.\ F^\theta{}_\lambda = \Sigma_\mu\ g_{\lambda\mu}.\ {}_\theta z^\mu$
$-\ k.\ g_{\lambda\mu}.\ F^{\theta\mu} = \Sigma_\mu\ g_{\lambda\mu}.\ {}_\theta z^\mu$
and get again:
$-\ k.\ F^{\theta\mu} = {}_\theta z^\mu$ (1.29.θ)
The contra-variant components of the ${}_\theta\mathbf{z}$ vectors are the θ-Th line of the EM field tensor [F].
That is we get the second important result of the present method, in extenso:
Any EM field expressed in a basis Ω of a 4-dimensional space possessing a symmetric fundamental tensor $T_2(.)(\Omega, \Omega)$ naturally supplies a set of four vectors ${}_\theta\mathbf{z}$ to express four splitting of the ewp $\mathbf{F}(M)\ \triangle\ d\mathbf{OM}/ds = {}_\theta[P].\ d\mathbf{OM}/ds + {}_\theta\mathbf{z}$ for $\theta = 0,1,2,3$ when and if the external force $\mathbf{F}(M)$ acting on a particle immerged in this EM field is reduced to the EM Lorentz force $\mathbf{F}_{EM}(M)$.

The "splitting" method explained here particularly fits to express the deep connection between any EM field and the local geometry. It will be interesting to analyze carefully all consequences of this paragraph and even more to explore the cases of other types of forces.

2.1.2. Consequences for the matrices:

Concerning our proposition for a description of the self-interaction we finally get two relations for $\theta = 0,1,2,3$:

$d\mathbf{OM}/ds \frown \mathbf{F}_{EM}(M) = {}_{\theta}[S].\ \mathbf{F}_{EM}(M) + {}_{\theta}\mathbf{s}$ (1.53)

$\mathbf{F}_{EM}(M) \frown d\mathbf{OM}/ds = {}_{\theta}[P].\ d\mathbf{OM}/ds + {}_{\theta}\mathbf{z}$ (1.54)

Recall that family F_1 satisfies:

$m^2.\ \Sigma_\lambda \Sigma_\mu \Sigma_\gamma \Sigma_\delta [\Sigma_\varepsilon g_{\gamma\varepsilon}.\ {}_{\theta}s_{\varepsilon\delta} - (\Sigma_\eta A_\eta{}^{\varepsilon}{}_{\delta}.\ u^\eta.\ g_{\gamma\varepsilon})].\ D_\lambda u^\gamma.\ D_\mu u^\delta.\ u^\lambda.\ u^\mu$
$+ m.\ \Sigma_\lambda \Sigma_\gamma \Sigma_\delta\ g_{\gamma\delta}.\ D_\lambda u^\gamma.\ {}_{\theta}s^\delta.\ u^\lambda.\ - {}_{\theta}\hat{c} = 0$

Because of (1.46), (1.49) and (1.36.1) we obtain:

$m^2.\ (k/c)^2.\ \Sigma_\lambda \Sigma_\mu \Sigma_\gamma \Sigma_\delta [\Sigma_\varepsilon g_{\gamma\varepsilon}.\ {}_{\theta}s_{\varepsilon\delta} - (\Sigma_\eta A_\eta{}^{\varepsilon}{}_{\delta}.\ u^\eta.\ g_{\gamma\varepsilon})].\ F^\gamma{}_\lambda.\ F^\delta{}_\mu.\ u^\lambda.\ u^\mu$
$+ m.\ (k/c).\ \Sigma_\lambda \Sigma_\gamma\ F^\gamma{}_\lambda.\ {}_{\theta}s_\gamma.\ u^\lambda.\ - {}_{\theta}\hat{c} = 0$

$(q/c)^2.\ \Sigma_\lambda \Sigma_\mu \Sigma_\gamma \Sigma_\delta [\Sigma_\varepsilon g_{\gamma\varepsilon}.\ {}_{\theta}s_{\varepsilon\delta} - (\Sigma_\eta A_\eta{}^{\varepsilon}{}_{\delta}.\ u^\eta.\ g_{\gamma\varepsilon})].\ F^\gamma{}_\lambda.\ F^\delta{}_\mu.\ u^\lambda.\ u^\mu$
$- (q/c).\ (\chi.\ c\ /\ m).\ \Sigma_\lambda \Sigma_\gamma\ F^\gamma{}_\lambda.\ \delta^\gamma{}_\theta.\ u^\lambda.\ - {}_{\theta}\hat{c} = 0$

$(q/c)^2.\ \Sigma_\lambda \Sigma_\mu \Sigma_\gamma \Sigma_\delta [\Sigma_\varepsilon g_{\gamma\varepsilon}.\ {}_{\theta}s_{\varepsilon\delta} - (\Sigma_\eta A_\eta{}^{\varepsilon}{}_{\delta}.\ u^\eta.\ g_{\gamma\varepsilon})].\ F^\gamma{}_\lambda.\ F^\delta{}_\mu.\ u^\lambda.\ u^\mu$
$- (q/c).\ (\chi.\ c\ /\ m).\ \Sigma_\lambda\ F^\theta{}_\lambda.\ u^\lambda.\ - \chi.\ du^\theta/ds = 0$

If we make use of the Lorentz current definition:

$j^\theta = q.\ u^\theta$ (1.55)

we get:

$(1/c)^2.\ \Sigma_\lambda \Sigma_\mu \Sigma_\gamma \Sigma_\delta [\Sigma_\varepsilon g_{\gamma\varepsilon}.\ {}_{\theta}s_{\varepsilon\delta} - (\Sigma_\eta A_\eta{}^{\varepsilon}{}_{\delta}.\ u^\eta.\ g_{\gamma\varepsilon})].\ F^\gamma{}_\lambda.\ F^\delta{}_\mu.\ j^\lambda.\ j^\mu$
$- (\chi\ /\ m).\ \Sigma_\lambda\ F^\theta{}_\lambda.\ j^\lambda.\ - \chi.\ du^\theta/ds = 0$

and at the end:

$(m/\chi.\ c^2).\ \Sigma_\lambda \Sigma_\mu \Sigma_\gamma \Sigma_\delta [\Sigma_\varepsilon g_{\gamma\varepsilon}.\ {}_{\theta}s_{\varepsilon\delta} - (\Sigma_\eta A_\eta{}^{\varepsilon}{}_{\delta}.\ u^\eta.\ g_{\gamma\varepsilon})].\ F^\gamma{}_\lambda.\ F^\delta{}_\mu.\ j^\lambda.\ j^\mu$
$- \Sigma_\lambda\ F^\theta{}_\lambda.\ j^\lambda = m.\ (du^\theta/ds)$ (1.56.θ)

Commentaries 04; quantification of atoms orbital and local geometry:

We remark that (1.56.θ = 0, 1, 2, 3) owns a formal analogy with the canonical formulation of the Yang-Mills actions [29; page 17; (5.1)]. That is we get a coherent expression for the relative force in Ω and can state that the linear part of it is effectively the Lorentz force.

As we could predict, the existence of a bilinear term should not be a surprise here and is related to the self-interaction (of the particle on itself). A moving charge, for example an electron around a proton can be considered as generating a current interacting with another current due to its own motion a few instant before (the two-body problem). That is an electron generates a magnetic field interacting with itself. In this sense, the bilinear term should be strongly connected with the magnetic induction forces inside the atom (of hydrogen, here).

Not only that, according to the usual formulation of this magnetic force and to the fact that the dimensions of an atom are quantified, we can now hope to get a formulation of the precise connections between the geometry of the basis and the quantified dimensions of the atom.

We just show now how we can get this intuition with the following short description.

$\Sigma_\varepsilon g_{\gamma\varepsilon}.\ {}_{\theta}s_{\varepsilon\delta} - (\Sigma_\eta A_\eta{}^{\varepsilon}{}_{\delta}.\ u^\eta.\ g_{\gamma\varepsilon})$

is the (γ, δ) term of the matrix

$\hat{G}.\ \{{}_{\theta}[S] - \Phi(\mathbf{u})\}$

That is the bilinear part of (1.56.θ = 0, 1, 2, 3) can be written:

$(m/\chi.\ c^2).\ j^*.\ \{\{[F]^t.\ \{\hat{G}.\ \{{}_{\theta}[S] - (1/q).\ \Phi(\mathbf{j})\}\}[F]\}\}.\ j\}$ (1.57) where we begin to work with a "spinor" representation of the vector current **j** (orange colour). Recall that in a very first and approximate formulation where we consider an electric charge dq instead of the charge q:

$j^\theta = dq.\ u^\theta = I.\ dt.\ u^\theta = (I/c).\ dx^\theta$

where I is the intensity of the electric current, and get from (1.56):

$(m.\ I^2 / \chi.\ c^4).\ dr^*.\ \{\{[F]^t.\ \{\hat{G}.\ \{_\theta[S] - (1/dq).\ \Phi(\mathbf{j})\}\}[F]\}\}.\ dr\}$

But in a very simple formulation (elementary physics), this force should have the following formalism:

$(\mu_0/2\pi.\ R).\ I^2.\ dL$

where dL is the length of and R the distance between the two considered electric trajectories, i. e. here two times the quantified rayon of an atom orbital a. We can now begin to guess the existence of a relation between the eigenvalues of the matrix $(m/\chi.\ c^4).\ \{\{[F]^t.\ \{\hat{G}.\ \{_\theta[S] - (1/dq).\ \Phi(\mathbf{j})\}\}[F]\}\}$ and the projection on the θ-axis of the basis of $(\mu_0/4\pi.\ a.\ dL)$. A more precise and exact connection will be the work for the next sections of this theory.

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