I. 1. Resume of the actual situation:

In the second part of the present work (Looking for masses) we began to explore a road connecting the Maxwell's force in vacuum (in a special and reduced formulation related to the fact that we introduced the (E) question and supposed for the latter a trivial solution) and the NSE.

Encouraged by the results obtained in the first part of our work (Foundations), we decided to *a priori* make use of the equation of state $p + \rho_0 = 0$ which is one possible solution of the equations of the general relativity accorded with the description of vacuum as "an energetic fluid" [37]; and we tried to explore in which way the Law of conservation concerning the EM energy in vacuum and the implicit hypothesis of the completeness of our universe (the relation 2 in Looking for masses) could induce the existence, in vacuum, of pre-Newtonian states ($\mathbf{\Gamma} = s. \nabla U + \mathbf{\Gamma}_0$), thus opening a way to connect usual considerations concerning electromagnetism on one side and gravity on the other side.

We discovered a possibility to interpret EM flows (associated to a field of speed vectors $\mathbf{V}$) as currents of pressure satisfying the GR condition $\Sigma_\alpha \Sigma_\beta.\ g_{\alpha\beta}.\ V^\alpha.\ V^\beta \approx 0$ in any part of a 4-D space with a not strongly changing metric, that is being solution of $\rho.\ (D\mathbf{V}/dt) = (\partial\mathbf{F}/\partial\tau) = \chi_0.\ T_2(°)(\partial, \mathbf{\Gamma}).\ \mathbf{\Gamma}$, if the field $\mathbf{V}$ is connected to the pre-Newtonian potential of gravity U with the following formula $V^\alpha = \chi_0.\ s^2.\ \Sigma_\gamma\ (\partial^2U/\partial y^\gamma\partial y^\alpha).\ (\partial U/\partial y^\gamma)$.

For coordinates of $\mathbf{V}$ oscillating between –c and + c, a special investigation concerning divergence-free pre-Newtonian fields with the reintroduction of the usual fact that we naturally consider the field of acceleration being the result of the derivation of the field of speed along the time ($\mathbf{\Gamma} = d\mathbf{V}/dt$) is automatically leading to a quantization of the phases of these oscillations.

At the end of this paragraph we must recall that we could won the initially equation of state ($p + \rho_0 = 0$) in making the hypothesis (see Foundations) that $d\mathbf{T}/dt = p(M, t)$. $\mathbf{V}$ and $\mathbf{T} = S.\ \partial\mathbf{F}(M, t)/\partial\tau$ where S was supposed to be the surface of the slice of a piece of elastic string on which $\mathbf{T}$ was making a traction.

The introduction of the picture of a classical elastic string in our model should not hurt for at least two good reasons:
1°) String theories are not only in trend but a necessity to try a connection with the Quantum physics;
2°) Our model is involving the Navier Stokes Equations (NSE) and foundations of these NSE are related to "balance equations" (see terminology of the Cambridge University*). One of these balance equations is the conservation of the total linear momentum in a given volume τ or in simpler form, Newton's second Law:

$d(\int_\tau \rho.\ \mathbf{v}.\ d\tau)/dt = \int \mathbf{T}.\ dS + \int_\tau \rho.\ \mathbf{f}.\ d\tau$

where t is the time, ρ the fluid density [masse/volume], $\mathbf{v}$ the fluid velocity [length/time], $\mathbf{T}$ the traction vector [force/area], $\mathbf{f}$ the body force density [force/mass].
This is suggesting that $\mathbf{T}$ is acting as a traction vector on each point M at any time t where the force $\partial\mathbf{F}/\partial\tau$ exists and that the respect of Newton's second Law doesn't avoid the introduction of a pictorial representation of a string associated with a particle. In our mind and in may be simpler words, $\partial\mathbf{F}/\partial\tau$ acts in deforming space-time so that a small peace of string appears that should be associated with a particle.

We now want to analyze this situation and see if we can induce more precise and more generalized results.

2. Analysing our intuition

2.1. Foundations of the demonstration itself

In despite of the fact that it is very difficult to summarize the way of reasoning that we followed, we can consider it as a circular path starting with one equation of state valid for vacuum:

$\rho_0 + p = 0$ (1)

We can also accept that this equation could hold in a space with any dimension N and that it would permit:

$(\rho_0 + p).\ {}^{(N)}\mathbf{V} = \mathbf{0}$ (2)

where ${}^{(N)}\mathbf{V}$ is the speed vector related to the energetic flow. *Per definition* of the divergence and *independently* of the underground geometry, this leads automatically to:

$\mathrm{div}[(\rho_0 + p).\ {}^{(N)}\mathbf{V}] = 0$ (3)

and to:

$\mathrm{div}\ (p.\ {}^{(N)}\mathbf{V}) = -\ \mathrm{div}\ (\rho_0.\ {}^{(N)}\mathbf{V})$ (4)

Coming back to our usual universe where we are living in, we leave here the N-dimensional space and reduce the discussion to the usual 4-D space of the GR.

$\mathrm{div}\ (p.\ {}^{(4)}\mathbf{V}) = -\ \mathrm{div}\ (\rho_0.\ {}^{(4)}\mathbf{V})$ (5)

Per definition of the divergence too, we always have the following identity:

$\Sigma_\beta\ V^\beta.\ [\partial p/\partial y^\beta] = \mathrm{div}\ (p.\ {}^{(4)}\mathbf{V}) - p.\ \mathrm{div}\ ({}^{(4)}\mathbf{V})$ (6)

that always will be reduced to:

$\Sigma_\beta\ V^\beta.\ [\partial p/\partial y^\beta] = \mathrm{div}\ (p.\ {}^{(4)}\mathbf{V})$ (7)

as long as one accept that the energetic flow could be compared with an incompressible liquid and so satisfies one of the NSE [35].

$\mathrm{div}\ ({}^{(4)}\mathbf{V}) = 0$ (8)

2.2. A special but important case:

The right hand term of (5) indicates that we have to think about the conservation of the $(\rho_0.\ {}^{(4)}\mathbf{V})$ vector quantity. Suppose that we could write:

$(\rho_0.\ {}^{(4)}\mathbf{V})$ = **invariant vector** (9)

This relation would be the equivalent relation, per unit of volume and up to a factor c², of the Law of conservation for the linear momentum.
The validity of (9) implies that the left term in the balance equation given above is zero. The total force acting on the surface S of our fixed mass (our small piece of string fixed at one of its two ends before its elongation) would equilibrate the total force acting over the volume of this mass.
This results, accordingly to the usual definition of any divergence, in:

$\mathrm{div}(\rho_0.\ {}^{(4)}\mathbf{V}) = (1/|g|^{1/2}).\ \Sigma_\alpha\ \partial(|g|^{1/2}.\ \rho_0.\ {}^{(4)}V^{\alpha})/\partial y^\alpha$
$\mathrm{div}(\rho_0.\ {}^{(4)}\mathbf{V}) = (1/|g|^{1/2}).\ \Sigma_\alpha\ \partial|g|^{1/2}/\partial y^\alpha\ .\ (\rho_0.\ {}^{(4)}V^{\alpha})$
$+\ \Sigma_\alpha\ \partial(\rho_0.\ {}^{(4)}V^{\alpha})/\partial y^\alpha$
$\mathrm{div}(\rho_0.\ {}^{(4)}\mathbf{V}) = (1/|g|^{1/2}).\ \Sigma_\beta\ \partial|g|^{1/2}/\partial y^\beta.\ (\rho_0.\ {}^{(4)}V^{\beta})$ (10)

where $|g|^{1/2}$ is the only quantity that can really vary independently of the invariant vector.

We now come back to the subject of our investigation which is to ask what would happen if the Maxwell's force $\partial\mathbf{F}(M, t)/\partial\tau$ could have the formalism $\chi_0.\ T_2(°)(\partial, \mathbf{\Gamma}).\ \mathbf{\Gamma}$ where $\mathbf{\Gamma}$ could itself own a pre-Newtonian formulation ($\mathbf{\Gamma} = s.\ \nabla U + \mathbf{\Gamma}_0$) and we calculate the infinitesimal quantity of work made by the motion of this Maxwell's force in the direction $(\delta\mathbf{r})$ and obtain:

$dC(M, t)$
$= \chi_0.\ s^2.\ \Sigma_\alpha\ \Sigma_\beta\ g_{\alpha\beta}.\ \{\Sigma_\gamma\ (\partial^2U/\partial y^\gamma\partial y^\alpha).\ (\partial U/\partial y^\gamma)\}.\ (d\mathbf{r})^\beta$
(11)

But because of the relation (1) and of general considerations in physics, we get in fact (see below):

$d(\partial W/\partial\tau) = -\ dp \equiv (\partial\mathbf{F}/\partial\tau).\ d\mathbf{r} = dC(M, t)$

and consequently:

$dp = -\chi_0.\ s^2.\ \sum_\alpha \sum_\beta g_{\alpha\beta}.\ \{\sum_\gamma (\partial^2 U/\partial y^\gamma \partial y^\alpha).\ (\partial U/\partial y^\gamma)\}.\ (\mathbf{dr})^\beta$ (12)

which is indirectly yielding:

$\partial p/\partial y^\beta = -\chi_0.\ s^2.\ \sum_\alpha g_{\alpha\beta}.\ \{\sum_\gamma (\partial^2 U/\partial y^\gamma \partial y^\alpha).\ (\partial U/\partial y^\gamma)\}$ (13)

and consequently:

$\sum_\beta V^\beta.\ \partial p/\partial y^\beta$

$= -\chi_0.\ s^2.\ \sum_\alpha \sum_\beta g_{\alpha\beta}.\ \{\sum_\gamma (\partial^2 U/\partial y^\gamma \partial y^\alpha).\ (\partial U/\partial y^\gamma)\}.\ V^\beta$ (14)

or because of (7):

$-\ \mathrm{div}\ (p.\ {}^{(4)}\mathbf{V})$

$= \chi_0.\ s^2.\ \sum_\alpha \sum_\beta g_{\alpha\beta}.\ \{\sum_\gamma (\partial^2 U/\partial y^\gamma \partial y^\alpha).\ (\partial U/\partial y^\gamma)\}.\ V^\beta$ (15)

Because of very general considerations made at the beginning of this paragraph (5), we can now write:

(10) – (15) = 0 (16)

$(1/|g|^{1/2}).\ \sum_\beta \partial|g|^{1/2}/\partial y^\beta.\ (\rho_0.\ {}^{(4)}V^{\beta})$

$-\ \chi_0.\ s^2.\ \sum_\alpha \sum_\beta g_{\alpha\beta}.\ \{\sum_\gamma (\partial^2 U/\partial y^\gamma \partial y^\alpha).\ (\partial U/\partial y^\gamma)\}.\ V^\beta = 0$

$\sum_\beta [(1/|g|^{1/2}).\ \partial|g|^{1/2}/\partial y^\beta.\ \rho_0$

$-\ \chi_0.\ s^2.\ \sum_\alpha g_{\alpha\beta}.\ \{\sum_\gamma (\partial^2 U/\partial y^\gamma \partial y^\alpha).\ (\partial U/\partial y^\gamma)\}].\ V^\beta = 0$ (17)

This is a very general relation that will always be valid under the conditions (1), (8) and (9) if the Maxwell's force and the field Γ have the formalism exposed before.

We think (*that is our intuition and our interpretation*) that this relation *is* the relation:

$\sum_\alpha \sum_\beta.\ g_{\alpha\beta}.\ V^\alpha.\ V^\beta = 0$ (18)

that must be satisfied by any streams in vacuum accordingly to the GR. This leads here to the main conclusion of our way of thinking:

$\sum_\alpha g_{\alpha\beta}.\ V^\alpha = [(\rho_0/|g|^{1/2}).\ \partial|g|^{1/2}/\partial y^\beta - \chi_0.\ s^2.\ \sum_\alpha g_{\alpha\beta}.\ \{\sum_\gamma (\partial^2 U/\partial y^\gamma \partial y^\alpha).\ (\partial U/\partial y^\gamma)\}]$ (19)

because this relation owns a formalism clearly connected with the (E) question as we will prove it a little bit later. The goal of following paragraphs and discussions is to test the validity of this intuition.

2.3. Physical context (Existence)

In the context of our discussion ($\rho_0.\ {}^{(4)}\mathbf{V}$) represents the extension ${}^{(4)}\mathbf{K}$ to a 4-D space of the generalized Poynting's vector $\mathbf{K}$ defined in the first part of this work. Accordingly to the fact that the 3-D part of this vector (see first part of our work) is defined with ($\mathbf{E} \wedge \mathbf{H}$) + **rot** $\mathbf{X}$, since one knows that the EM field owns oscillating solutions, it must be the same for ($\mathbf{E} \wedge \mathbf{H}$) and an invariance of $\mathbf{K}$ could only occur if **rot** $\mathbf{X}$ would equilibrate these oscillations at every moment.

Otherwise one also knows that any observer at rest at the origin of an inertial frame should measure $<\mathbf{E} \wedge \mathbf{H}> = \mathbf{0}$. In this case, for an observation made over a "long enough" time (for the observer) one could effectively say that $\mathbf{K}$ has an average equal to < **rot** $\mathbf{X}$ > which would only be invariant if the average of the curl vector is invariant over the same period of time. In the special case where the average of the curl vector is invariant and equal to zero, the preceding discussion only makes sense in considering a succession of "short duration invariant" vectors of which the sum over a relatively small period of time (for the observer) is zero. As one see, we don't refuse to consider the vector $\mathbf{X}$ in the discussion because it plays an important role.

3. Extension of the demonstration to any force

3.1. Principle of the extension:

We now make the demonstration for any volumetric density of force in vacuum again; this simply means that instead of $\sum_\gamma (\partial^2 U/\partial y^\gamma \partial y^\alpha).\ (\partial U/\partial y^\gamma)$ we just have to write $(\partial \mathbf{F}(M, t)/\partial \tau)^\alpha = \partial F^\alpha(M, t)/\partial \tau$ without taking care of the real nature of the force involved in the discussion. Since a volumetric density of force always is a force per unit of volume, the work of this volumetric density of force along any path is a force per unit of surface, that is a pressure. This is the fundamental reason why we could write $dC(M, t) = (\partial \mathbf{F}/\partial \tau).\ \mathbf{dr} \equiv -\ dp$ (3.1) in § 2.1. page 2. Our way of thinking is based on the method of the work integral (see [33] page 103, for an example and the definition). The result is:

$dp = -\ \sum_\alpha \sum_\beta g_{\alpha\beta}.\ \partial F^\alpha(M, t)/\partial \tau.\ (\mathbf{dr})^\beta$ (3.2)

where we take care of the definition of the inner dot product in a 4-dimensional space of which the geometry is not necessary exactly flat and Euclidian, including de facto the extension of this discussion within the world of the GR, considering the Minkowski's space as a special case. If we suppose that we are authorized to consider (3.2) as a Taylorisation up to the first order of the pressure p, than we can write, at least approximately:

$\partial p/\partial y^\beta = -\ \sum_\alpha g_{\alpha\beta}.\ \partial F^\alpha(M, t)/\partial \tau$ (3.3)

and consequently:

$\sum_\beta V^\beta.\ \partial p/\partial y^\beta = -\ \sum_\alpha \sum_\beta g_{\alpha\beta}.\ \partial F^\alpha(M, t)/\partial \tau.\ V^\beta$ (3.4)

We now consider the very interesting case of an invariant volumetric density of energy ρ_0 and continue this extension of our demonstration with (24):

$\mathrm{div}(\rho_0.\ {}^{(4)}\mathbf{V}) = (\rho_0/|g|^{1/2}).\ \sum_\alpha \partial|g|^{1/2}/\partial y^\alpha.\ {}^{(4)}V^{\alpha}$

$+\ \rho_0.\ \sum_\alpha \partial({}^{(4)}V^{\alpha})/\partial y^\alpha$ (24)

This yields some consequences: *"If vacuum is a kind of incompressible energetic "fluid" (8), the volumetric density of force ∂**F**(M, t)/∂τ and the speed of the energetic flow **V** always are orthogonal"*.

Demonstration:

$\rho_0 = \text{invariant}$

⇓

$\mathrm{div}(\rho_0.\ {}^{(4)}\mathbf{V}) = \rho_0.\ \mathrm{div}(\ {}^{(4)}\mathbf{V})$

because of the definition of any divergence

⇓

$\mathrm{div}(\rho_0.\ {}^{(4)}\mathbf{V}) = 0$ because of (8)

⇓

$\mathrm{div}\ (p.\ {}^{(4)}\mathbf{V}) = 0$ because of (5)

⇓

$\sum_\beta V^\beta.\ [\partial p/\partial y^\beta] = 0$ because of (6), (7) and (8)

⇓

$\sum_\alpha \sum_\beta g_{\alpha\beta}.\ \partial F^\alpha(M, t)/\partial \tau.\ V^\beta = 0$ because of (3.4)

⇓

$\partial \mathbf{F}(M, t)/\partial \tau.\ \mathbf{V} = 0$ (3.5)

Even if vacuum is a kind of compressible energetic "fluid", the volumetric density of force $\partial\mathbf{F}(M,\ t)/\partial\tau$ and the speed of the energetic flow $\mathbf{V}$ always are orthogonal. Demonstration:

$$\rho_0 = \text{invariant}$$
$$\Downarrow$$
$$\mathrm{div}(\rho_0.\ {}^{(4)}\mathbf{V}) = \rho_0.\ \mathrm{div}(\ {}^{(4)}\mathbf{V})$$
because of the definition of any divergence
$$\Downarrow$$
$$\mathrm{div}\ (p.\ {}^{(4)}\mathbf{V}) = -\ \rho_0.\ \mathrm{div}(\ {}^{(4)}\mathbf{V}) \text{ because of (5)}$$
$$\Downarrow$$
$$\Sigma_\beta\ V^\beta.\ [\partial p/\partial y^\beta] + p.\ \mathrm{div}\ ({}^{(4)}\mathbf{V}) = -\ \rho_0.\ \mathrm{div}(\ {}^{(4)}\mathbf{V})$$
because of (6)
$$\Downarrow$$
$$\Sigma_\beta\ V^\beta.\ [\partial p/\partial y^\beta] + p.\ \mathrm{div}\ ({}^{(4)}\mathbf{V}) = p.\ \mathrm{div}(\ {}^{(4)}\mathbf{V})$$
because of (1)
$$\Downarrow$$
$$\Sigma_\alpha\ \Sigma_\beta\ g_{\alpha\beta}.\ \partial F^\alpha(M,\ t)/\partial\tau.\ V^\beta = 0 \text{ because of (3.4)}$$
$$\Downarrow$$
$$\partial\mathbf{F}(M,\ t)/\partial\tau.\ \mathbf{V} = 0 \quad (3.5)$$

These results are suggesting that the orthogonally between the volumetric density of force $\partial\mathbf{F}(M,\ t)/\partial\tau$ and the speed of the energetic flow $\mathbf{V}$ always is a "signature" of the invariance of the volumetric density of energy. If stable states of any particle, as we think, always are associated with an invariant volumetric density of energy, this is suggesting that particles in a stable state are moving along paths where the orthogonally is respected. Is this suggestion correct?

Let us now consider the case of any volumetric density of energy, that is eventually changing. Demonstration:

$$\text{Any } \rho_0$$
$$\Downarrow$$
$$\mathrm{div}(\rho_0.\ {}^{(4)}\mathbf{V}) = (1/|g|^{1/2}).\ \Sigma_\alpha\ \partial(|g|^{1/2}.\ \rho_0.\ {}^{(4)}V^{\alpha})/\partial y^\alpha$$
$$= \rho_0.\ \mathrm{div}(\ {}^{(4)}\mathbf{V}) + \Sigma_\alpha\ \partial\rho_0/\partial y^\alpha.\ {}^{(4)}V^{\alpha}$$
because of the definition of any divergence
$$\Downarrow$$
$$\mathrm{div}\ (p.\ {}^{(4)}\mathbf{V}) = -\ \mathrm{div}(\rho_0.\ {}^{(4)}\mathbf{V}) \text{ because of (5)}$$
$$\Downarrow$$
$$\Sigma_\beta\ V^\beta.\ [\partial p/\partial y^\beta] + p.\ \mathrm{div}\ ({}^{(4)}\mathbf{V})$$
$$=$$
$$-\ \rho_0.\ \mathrm{div}(\ {}^{(4)}\mathbf{V}) - \Sigma_\alpha\ \partial\rho_0/\partial y^\alpha.\ {}^{(4)}V^{\alpha}$$
because of (6)
$$\Downarrow$$
$$\Sigma_\beta\ V^\beta.\ [\partial p/\partial y^\beta] = -\ \Sigma_\alpha\ \partial\rho_0/\partial y^\alpha.\ {}^{(4)}V^{\alpha}$$
because of (1)
$$\Downarrow$$
$$\Sigma_\alpha\ \Sigma_\beta\ g_{\alpha\beta}.\ \partial F^\alpha(M,\ t)/\partial\tau.\ V^\beta = -\ \Sigma_\alpha\ \partial\rho_0/\partial y^\alpha.\ {}^{(4)}V^{\alpha}$$
because of (3.4)
$$\Downarrow$$
$$\partial\mathbf{F}(M,\ t)/\partial\tau.\ \mathbf{V} = -\ \Sigma_\alpha\ \partial\rho_0/\partial y^\alpha.\ {}^{(4)}V^{\alpha} \qquad (3.6)$$

The inner (dot) product in vacuum (equation of state 1) of the volumetric density of force $\partial\mathbf{F}(M,\ t)/\partial\tau$ and the speed of the energetic flow $\mathbf{V}$ for any volumetric density of energy is given by (3.6) and is not necessary zero.

Let us now consider the case of any invariant volumetric density of linear momentum. Demonstration:

$$\text{Any } \rho_0.\ {}^{(4)}\mathbf{V} \text{ invariant}$$
$$\Downarrow$$
$$\mathrm{div}(\rho_0.\ {}^{(4)}\mathbf{V}) = (1/|g|^{1/2}).\ \Sigma_\alpha\ \partial(|g|^{1/2}.\ \rho_0.\ {}^{(4)}V^{\alpha})/\partial y^\alpha$$
$$= (\rho_0\ /\ |g|^{1/2}).\ \Sigma_\beta\ (\partial|g|^{1/2}/\partial y^\beta).\ {}^{(4)}V^{\beta} \qquad (10)$$
because of the definition of any divergence
$$\Downarrow$$
$$\mathrm{div}\ (p.\ {}^{(4)}\mathbf{V}) = -\ (\rho_0\ /\ |g|^{1/2}).\ \Sigma_\beta\ (\partial|g|^{1/2}/\partial y^\beta).\ {}^{(4)}V^{\beta} \text{ because of (5)}$$
$$\Downarrow$$
$$\Sigma_\beta\ V^\beta.\ [\partial p/\partial y^\beta] + p.\ \mathrm{div}\ ({}^{(4)}\mathbf{V})$$
$$=$$
$$-\ (\rho_0\ /\ |g|^{1/2}).\ \Sigma_\beta\ (\partial|g|^{1/2}/\partial y^\beta).\ {}^{(4)}V^{\beta}$$
because of (6)
$$\Downarrow$$
$$\Sigma_\beta\ V^\beta.\ [\partial p/\partial y^\beta] + \Sigma_\alpha\ \partial\rho_0/\partial y^\alpha.\ {}^{(4)}V^{\alpha} = \rho_0.\ \mathrm{div}\ ({}^{(4)}\mathbf{V})$$
because of (1)
$$\Downarrow$$
$$\Sigma_\alpha\ \Sigma_\beta\ g_{\alpha\beta}.\ \partial F^\alpha(M,\ t)/\partial\tau.\ V^\beta + \Sigma_\alpha\ \partial\rho_0/\partial y^\alpha.\ {}^{(4)}V^{\alpha}$$
$$=$$
$$\rho_0.\ \mathrm{div}\ ({}^{(4)}\mathbf{V})$$
because of (3.4)
$$\Downarrow$$
$$\Sigma_\beta\ [\Sigma_\alpha\ g_{\alpha\beta}.\ \partial F^\alpha(M,\ t)/\partial\tau + \partial\rho_0/\partial y^\beta].\ {}^{(4)}V^\beta$$
$$=$$
$$\rho_0.\ \mathrm{div}\ ({}^{(4)}\mathbf{V})$$
$$(3.7)$$

As said just before, the inner (dot) product in vacuum (equation of state 1) of the volumetric density of force $\partial\mathbf{F}(M,\ t)/\partial\tau$ and the speed of the energetic flow ${}^{(4)}\mathbf{V}$ for any volumetric density of energy is given by (3.6) and is not necessary zero. For any invariant $\rho_0.\ {}^{(4)}\mathbf{V}$, the volumetric density of energy and the speed of the flow can vary. Thus in this case, we are in the preceding one concerning the volumetric density of energy; that is the first part of the sum on the left hand term of (3.7) is not necessary zero. One must remark that no other hypothesis than (1) and the invariance of $\rho_0.\ {}^{(4)}\mathbf{V}$ has been made to obtain (3.7). In particularly we didn't consider the question of the compressibility of the vacuum. If (8) is valid, (3.7) is reduced in $\Sigma_\beta\ [\Sigma_\alpha\ g_{\alpha\beta}.\ \partial F^\alpha(M,\ t)/\partial\tau + \partial\rho_0/\partial y^\beta].\ {}^{(4)}V^\beta = 0$ (3.8). Note also that if ρ_0 is "momentary" invariant, then (3.7) is "momentary" reduced in (8) because of the way of thinking leading to (3.5).

Because of the definition of any divergence:

$$\rho_0.\ \mathrm{div}\ ({}^{(4)}\mathbf{V}) = (\rho_0\ /\ |g|^{1/2}).\ \Sigma_\alpha\ \partial(|g|^{1/2}.\ {}^{(4)}V^{\alpha})/\partial y^\alpha$$
$$=$$
$$\rho_0.\ \Sigma_\alpha\ \partial^{(4)}V^{\alpha}/\partial y^\alpha + (\rho_0.\ {}^{(4)}V^{\alpha}/\ |g|^{1/2}).\ \Sigma_\alpha\ \partial|g|^{1/2}/\partial y^\alpha \ (3.9)$$

The invariance of $\rho_0.\ \mathbf{V}$ implies, in the coordinates language:

$$\Sigma_\alpha\ \partial\rho_0/\partial y^\alpha.\ {}^{(4)}V^\alpha + \rho_0.\ \Sigma_\alpha\ \partial^{(4)}V^{\alpha}/\partial y^\alpha = 0 \ (3.10)$$

Also (3.7) is transformed in:

$$\Sigma_\beta\ [\Sigma_\alpha\ g_{\alpha\beta}.\ \partial F^\alpha(M,\ t)/\partial\tau + \partial\rho_0/\partial y^\beta].\ {}^{(4)}V^\beta$$
$$=$$
$$\rho_0.\ \Sigma_\alpha\ \partial^{(4)}V^{\alpha}/\partial y^\alpha + (1/|g|^{1/2}).\ \Sigma_\alpha\ (\rho_0.\ {}^{(4)}V^{\alpha}).\ \partial|g|^{1/2}/\partial y^\alpha$$
because of (3.9)
$$\Downarrow$$
$$\Sigma_\beta\ [\Sigma_\alpha\ g_{\alpha\beta}.\ \partial F^\alpha(M,\ t)/\partial\tau + \partial\rho_0/\partial y^\beta].\ {}^{(4)}V^\beta$$
$$=$$
$$-\ \Sigma_\alpha\ \partial\rho_0/\partial y^\alpha.\ {}^{(4)}V^\alpha + (1/|g|^{1/2}).\ \Sigma_\alpha\ (\rho_0.\ {}^{(4)}V^{\alpha}).\ \partial|g|^{1/2}/\partial y^\alpha$$
because of (3.10)
$$\Downarrow$$
$$\Sigma_\beta\ [\Sigma_\alpha\ g_{\alpha\beta}.\ \partial F^\alpha(M,\ t)/\partial\tau + 2.\ \partial\rho_0/\partial y^\beta].\ {}^{(4)}V^\beta$$
$$=$$
$$(1/|g|^{1/2}).\ \Sigma_\alpha\ (\rho_0.\ {}^{(4)}V^{\alpha}).\ \partial|g|^{1/2}/\partial y^\alpha \ (3.11)$$

Our *intuition and hypothesis* is to interpret equation (3.7) and consequently (3.11) as the equation of the motion for "something (a particle or a state of a particle)" associated with ρ_0, thus as a solution of the actual generalization of Newton's Equations inside the GR [17 with $ds = c.\ dt$ (3.12)]:

$$\rho_0.\ d^2y^\theta/ds^2 + \rho_0.\ \Sigma_\gamma\ \Sigma_\delta\ \Gamma_\gamma{}^\theta{}_\delta\ .\ (dy^\gamma/ds).\ (dy^\delta/ds) = (1/c^2).\ \partial F^\theta/\partial\tau \qquad (3.13)$$

when $\mathbf{v} = \mathbf{V}$ (3.14)

3.2. Implications of our proposition:

3.2.1. First consequence:

This hypothesis implies for each motion the invariance of the right hand term in (3.11). As we still supposed that $\rho_0 .\ {}^{(4)}\mathbf{V} = {}^{(4)}\mathbf{K}$ is an invariant vector to establish this relation, then our hypothesis only implies:

$$\begin{gathered}(1/|g|^{1/2}).\ \Sigma_\alpha\ \partial(|g|^{1/2}.\ {}^{(4)}K^{\alpha})/\partial y^{\alpha} \\ = \\ \text{invariant for a given motion}\end{gathered}$$

that can also be traduced in:

$$\begin{gathered}(1/|g|^{1/2}).\ \text{Trace of } T_2\,(°)(\partial,\ |g|^{1/2}.\ {}^{(4)}\mathbf{K}) \\ = \\ \text{invariant for a given motion } (3.15)\end{gathered}$$

In despite of the fact that an hypothesis is not a demonstration, the invariance of a trace is an encouraging condition appearing in our model because we know that the trace of a matrix is preserved when one is changing from a basis to another one for certain families of basis transformations (We will explore this point later).

That is if the signature of the motion for a given particle in a given family $\Im$ of basis characterized by the invariance of $|g|$ would be the Trace of the matrix $T_2\,(°)(\partial,\ |g|^{1/2}.\ {}^{(4)}\mathbf{K})$, then this particle would have the same signature in all basis of $\Im$; thus explaining why two observers based at the origin of two different frames, both belonging to $\Im$, would recognize the same particle. At the end our hypothesis leads to:

$$\begin{gathered}\text{In a given family } \Im \text{ of metrics with invariant } |g| \\ \text{Trace of } T_2\,(°)(\partial,\ {}^{(4)}\mathbf{K}) = 0 \\ \text{invariant for any given motion in vacuum with any } \rho_0 .\ {}^{(4)}\mathbf{V} \\ \text{invariant } (3.16)\end{gathered}$$

The invariance of the volumetric density of linear momentum in any family of metrics characterized by $|g|$ and its invariance leads to an invariant value equal to zero for the trace of the matrix $T_2(°)(\partial,\ {}^{(4)}\mathbf{K})$.

3.2.2. Second consequence:

The next consequence of our hypothesis is finally:

$[\Sigma_\alpha\ g_{\alpha\beta}.\ \partial F^{\alpha}(M, t)/\partial\tau + 2.\ \partial\rho_0/\partial y^{\beta}] \equiv \propto.\ \Sigma_\beta\ g_{\alpha\beta}.\ V^{\beta}$ (3.17)

Yielding for invertible fundamental metrics only the following astonishing relation:

$\partial^{(4)}\mathbf{F}(M, t)/\partial\tau + 2.\ [g]^{-1}.\ {}^{(4)}\nabla\rho_0 \equiv \propto.\ {}^{(4)}\mathbf{V}$ (3.18)

Where the symbol $\propto$ appears here to pay attention to the coherence concerning the physical units.

Important questions have to be discussed and addressed in our model:

1°) The distinction between the speed **V** of the energetic flow and the speed **v** of anything which is not necessary the energetic flow (like the swimmer in the stream is not always exactly following the stream if physical circumstances allow it); a precision concerning this distinction will certainly introduce a modification of the identification between (3.11) and (18). The latter is written and valid for **v** and not for **V** within the GR; that is our identification only holds for swimmers following the stream corresponding to the hypothesis: **v** = **V** (3.14) made above. It is important to find the exact connection between **v** and **V** for any other particular situation in the nature to get a correct and still more generalized identification between (3.11) and (18).

2°) Regions of the universe with invertible metrics allow the confrontation between (3.13) and (3.18). As said just before in 1°), this one only holds here for the very special case **v** = **V** but leads, because of this condition of validity, to a surprising equation:

$$\begin{gathered}½.\ [g].\ \{\rho_0 .\ dV^{\theta}/ds + \rho_0 .\ \Sigma_\gamma\ \Sigma_\delta\ \Gamma_{\gamma}{}^{\theta}{}_{\delta} .\ V^{\gamma}.\ V^{\delta} - \propto (1/c^2).\ {}^{(4)}V^{\theta}\} \\ \equiv \\ - \propto.\ \{{}^{(4)}\nabla\rho_0\}^{\theta}\ (3.19)\end{gathered}$$

connecting the coordinates of the speed (of the flow), its first derivate along the path, the connections coefficients (Christoffel's symbols), the coefficients of the metric on one side and the gradient of the volumetric density of energy on the other side.

This equation owns enormous consequences:

For example, *in a flat region of space-time with invariant volumetric density of energy*, we should write it:

$\rho_0 .\ dV^{\theta}/ds - \propto (1/c^2).\ {}^{(4)}V^{\theta} = 0$ (3.20)

Before going further and in order to get a correct analyze concerning this latter, we must find what exactly is this $\propto$ symbol. A rapid analyze built on (3.14), (3.18) and on the fact that we could theoretically write:

$\partial\mathbf{F}/\partial\tau = [\partial\rho/dt + \rho.\ \partial(d\tau/dt)/\partial\tau].\ \mathbf{v} + \rho.\ (d\mathbf{v}/dt)$ (3.21)

where ρ is the volumetric density of matter [kg/m³ in the MKSA system] leads to the first impression that:

$\propto\ \approx [\partial\rho/dt + \rho.\ \partial(d\tau/dt)/\partial\tau]$ (3.22)

so that our unknown symbol $\propto$ should be related to the variations along the time of the volumetric density of mass (physical units: kg/second x length³) and to some variations also along the time of the considered volumes too (Do we have here a relation with the problem of the dilatation-inflation…?).

Thus, for the case of an invariant volumetric density of energy in a flat region of the universe, (3.20) is reduced in [because of the equivalence established within the SR: $\rho_0 = \rho.\ c^2$ (3.23)]:

$dV^{\theta}/ds - (1 / c^4).\ \partial(d\tau/dt)/\partial\tau.\ {}^{(4)}V^{\theta} = 0$ (3.24)

Except if this region underlies a very strong inflation (to counterbalance the great value of c^4), one can expect that, most part of the time, it will be reduced in:

$dV^{\theta}/ds = 0$ (3.25)

meaning that *the energetic flow has an invariant speed*.

If we try to understand this condition (3.24) in more general circumstances and in recalling the baryonic law of conservation: $(d\tau/dt) = (\text{div}\ \mathbf{V}).\ \tau$ [20; page 558; (22.2)] where **V** is the 4-velocity of the fluid (here the local flow of energy), when the divergence of this velocity is not directly depending of the volume itself, we can calculate in a first approximation:

$\partial(d\tau/dt)/\partial\tau \approx \text{div}\ \mathbf{V} + 0(2)$ (3.26)

leading again to the problem of the compressibility of vacuum which was not taken in consideration to win (3.7) and consequently (3.11). *Clearly, the incompressibility of vacuum, (= validity of equation 8), in a context respecting the baryonic law of conservation would give $d\tau/dt = 0$ and (3.25) again.* It seems to be a coherent result in accordance first with the start hypothesis of an invariant volumetric density of linear momentum and second with the reality since we know that the light is propagating straight forward in the regions of the universe belonging a flat geometry as long as nothing is disturbing it (f. ex: a gravitational field).

Otherwise, if we would have no insurance concerning the incompressibility of vacuum, (= 8 is not necessary valid) but if the baryonic law of conservation would hold, we would always have:

$dV^{\theta}/ds - (1 / c^4).\ \text{div}\ \mathbf{V}.\ {}^{(4)}V^{\theta} = 0$ (3.27)

The derivation of (3.27) along the path gives:
d^2V^θ/d^2s
$= (1 / c^4).\ d(\mathrm{div}\ \mathbf{V})/ds.\ {}^{(4)}V^\theta + (1 / c^4).\ \mathrm{div}\ \mathbf{V}.\ {}^{(4)}dV^\theta /ds$
$= (1 / c^4).\ [d(\mathrm{div}\ \mathbf{V})/ds + (1 / c^4).\ (\mathrm{div}\ \mathbf{V})^2].\ {}^{(4)}V^\theta$ (3.28)
clearly leading to oscillating solutions for the speed **V** of the energetic flow. The relation (3.27) is also suggesting that, at least in a flat region of the universe, a deviation of the energetic flow (theoretically equivalent with d**V**/dt non zero) is entirely connected with the local property of compressibility. A local compressibility allows a deviation.

Another consequence of this comparison – in any region of the universe- between (3.18) and (3.21), if allowed by the logic, would be:
$2.\ [g]^{-1}.\ {}^{(4)}\nabla\rho_0 \equiv \rho.\ (d\mathbf{V}/dt)$ (3.29)
or because of (3.23):
$(1/\rho_0).\ [g]^{-1}.\ {}^{(4)}\nabla\rho_0 \equiv \tfrac{1}{2}.\ (1/c^2).\ (d\mathbf{V}/dt)$
absolutely in accordance with (3.25) when the volumetric density of energy is invariant.
${}^{(4)}\nabla\rho_0 = 0 \Rightarrow d\mathbf{V}/dt = \mathbf{0}$ (3.30)
The equation (3.29) is interesting because introducing an extension concerning the inertial regions (IR); these IR must satisfy:
$[g]^{-1}.\ {}^{(4)}\nabla\rho_0 = \mathbf{0}$ (3.31)
and are not automatically reduced to the regions with invariant volumetric density of energy.

Just a remark:
Remember that we got in a 3-D flat space with the help of Maxwell's Laws <u>and for trivial solutions of the (E) question</u>:
$\partial(\varepsilon_0.\mu_0.\ \rho_0.\mathbf{V})/\partial t = \varepsilon_0.\ T_2(^\circ)(\partial, \mathbf{E}).\ \mathbf{E} + \mu_0.\ T_2(^\circ)(\partial, \mathbf{H}).\ \mathbf{H} - {}^{(3)}\nabla\rho_0 + \varepsilon_0.\ \mu_0.\ \partial(\mathbf{rot\ X})/\partial t$
Let us compare with (3.18) that one can write in a 3-D flat space:
$\partial^{(3)}\mathbf{F}_{\text{applied}}(M, t)/\partial\tau + 2.\ {}^{(3)}\nabla\rho_0 \equiv \propto.\ {}^{(3)}\mathbf{V}$ (3.32)
We finally get, if the applied force is $\partial(\varepsilon_0.\mu_0.\ \rho_0.\mathbf{V})/\partial t$:
$\varepsilon_0.\ T_2(^\circ)(\partial, \mathbf{E}).\ \mathbf{E} + \mu_0.\ T_2(^\circ)(\partial, \mathbf{H}).\ \mathbf{H} + \varepsilon_0.\ \mu_0.\ \partial(\mathbf{rot\ X})/\partial t + {}^{(3)}\nabla\rho_0 \equiv \propto.\ {}^{(3)}\mathbf{V}$

3.2.3. Third consequence : the Lagrangian in our model.

I. Introduction:
An important item was until here not discussed in this work. We mean the first law of thermodynamics, sometimes called the local law of energy conservation [see 20; page 692; (26.10)]. This law, because of the relation (1) which is a foundation of our approach, is leading to:
$\Delta\rho_0 = 0$
$\delta\rho_0 = -\ \xi.\ \rho'_0$ (initial value)

In fact we should remember that (1) is the extreme case of a statistical law $<\rho_0 + p> = 0$ that we obtained in the first part of our approach introducing the pictorial idea of a string elongation to describe the behaviour of the geometry in vacuum [see Foundations].

We should also recall that:
$\mathbf{d}\rho_0 = n.\ T.\ \mathbf{ds}$ [see 20; page 560; (22.6)]
$(\partial\rho_0/\partial n)_s = 0$ [see 20; page 560; (22.7a)]
$(\partial\rho_0/\partial s)_n = n.\ T$ [see 20; page 560; (22.7b)]
where ρ_0 is the total mass-energy (including rest mass, thermal energy, compressional energy, etc) contained in a unit three dimensional volume of the rest frame, n is the baryon number density (i.e. the number of baryons per unit three-dimensional volume of rest frame, with anti baryons -if any- counted negatively), s is the entropy per baryon in rest frame (n. s is the entropy per unit volume) and T the temperature in rest frame.

One also have:
$\nabla_w\rho_0 = n.\ T.\ <\mathbf{ds}, \mathbf{w}>$
which is giving the changes along a given direction in the fluid (along a given tangent vector **w**). To resume these well known results valid in any three dimensional volume, the changes of the volumetric density of total energy eventually present in vacuum only depend on the baryon number density, on the temperature (which is known to be approximately 2.7°K, in average) and on the changes concerning the entropy per baryon.

But our approach is in fact involving some more new ideas that can be a little bit compared with the new formulation of the SCC [30]. What do we mean with this? That the main preoccupation of our exploration is to scrutinize the hypothesis if the geometric structure of space time could be itself the source for the birth of particles (baryons f. e.) in special circumstances which have to be determined. Until here we only have consider vacuum as an energetic fluid with currents of pressure. We suggest that some of these currents have to be related with elementary particles of the modern physics (see Looking for masses). Our approach is consequently problematic because telling the difficult question: how can we really count the number of baryons in a given volume? To realize it implies not only to own a clear method to do it but also a clear definition of what a baryon (and more generally a particle) is. Since we know the duality of waves and particles (and quite more terrible the possibility of a dual state between wave and particle: see the last experiment of Afshar), this question will have no evident and trivial answer at all.

We find the beginning of an explanation in considering a concrete example. Let us consider underwater streams coming out from the walls all around of a swimming pool. In the direct vicinity of a stream, it seems to be hard, strong and one can "feel" its form with the hand. In the middle of the bath one can no more recognize its existence. We suggest that it is a good picture to represent what a particle is: a stream, sometimes well organized, sometimes scattering.

To finish this provisory introduction concerning the definition of a Lagrangian for our model, we will refer to the book of Wheeler, Misner and Thorne [20; page 564]: "The pressure gradient, not "gravity", is responsible for all deviation of flow lines from geodesics."

On progress…

3.3. Current of pressure equal particle?

The goal of this paper is to analyze in which way and how far we could compare any particle of the physics with a current of pressure.

At human usual scale, it is well known that any piece of matter occupies a three dimensional volume and presents one surface orthogonally positioned relatively to any motion that it can do in a three dimensional space. At any moment, even if at rest in the frame of the observer, it's offering its complete external surface as bound against a (not so) "virtual" travel through the time. We can resume this reality, taking in fact care of the fact that any material object has a life with two limits (the birth and the death), in saying that these objects always exert a pressure, at least in the temporal dimension when they are at rest. This idea would approximately have the following formalism:
(the pressure in the temporal dimension) = (unknown force against a travel in the time)/(total three dimensional surface of the body)

But let us now leave this deep thoughts and come back to more concrete considerations. We know that any motion of a material body occurring in a fluid or in a gas will be retard because of the friction. The result for a free falling down shark in the see is a speed limit. The Theory of the Relativity (A. Einstein) offers two informations: i) Independently of the energy that we will spend to increase the speed of the body, we will never get a speed higher than c; the speed of light in vacuum and ii) Vacuum is no ether; the speed limit also exists for motions occurring in … nothing (accordingly to the representation of the world that this Theory develops). This is strongly suggesting (if one accept a mental comparison with the shark in the see) that vacuum also behaves like a kind of ocean and owns a certain coefficient of elasticity leading to this speed limit. Since we know that vacuum is the total opposite of classical matter (Morley-Michelson Experiment), we are obliged to accord this elasticity to the fields that are supposed to be present in the volumes occupied by vacuum.

Our representation of vacuum gets slowly a coherent face. And this face could be a connection between the GR (Generalized Theory of Relativity) and the QT (Quantum Theory). Effectively, let us interpret the relation [$\partial\mathbf{F}(M, t)/\partial\tau . \mathbf{V} = 0$ (3.5)] as one of the (post-Newtonian) law of the geometric optics in curved space time: "Amplitude (or polarization) is perpendicular to wave vector" [see 20 page 573; (22.28) and (22.28')].

This is possible if we accept that our energetic stream moving at **V** speed is moving like a ray in the direction of the wave vector **k** parallel to **V**; and if we decide to believe that polarization vector and volumetric density of force in vacuum are parallel vectors [$\partial\mathbf{F}(M, t)/\partial\tau \,//\, \mathbf{f}$ = polarization vector].

This is an interesting proposition if one remember the very beginning of our investigations [see Foundations.pdf]. It means that spontaneous fluctuations in vacuum could be the origin of local distortions of the local geometry and of polarizations; or that spontaneous fluctuations in vacuum are polarization forces.

Next step of our theory will be to confront the fundamental equations of the geometric optics with this interpretation of our own equations. As one can guess it now, we will encounter difficulties concerning the long distance action of our interpretation; we will explore that later.

4. When are we allowed to write a force under the Maxwell's formalism ?

4.1. Definition of the "Maxwell's formalism"

In our mind any force (electric, magnetic, …) per unit of volume with following formalism

$$\partial^{(4)}\mathbf{F}/\partial\tau = y_0 . T_2(°)(\partial, {}^{(4)}\mathbf{Y}) . {}^{(4)}\mathbf{Y} \qquad (4.1)$$

is a "Maxwell's force" corresponding to the field **Y**.

4.2. Formalism:

The question is: "Can any force be written under this Maxwell's formalism ?" This question is equivalent to the other one: "Is there always at least one frame where any force could be written under the form (3.1) ?" To answer, one have to know the general expression of "any" force. The common definition of a force is the derivate of the linear momentum by respect for the time:

$\mathbf{F} = d\mathbf{p}/dt$

$\mathbf{p} = m. \mathbf{v}$

On progress…

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Bibliography:

[17] Allgemeine Relativitäts- Theorie ;Torsten Fließbach ; 4. Auflage; Spektrum der Wissenschaft; 2003

[20] Misner, Thorne et Wheeler : Gravitation ; 1973.

[30] Self Creation Cosmology An Alternative Gravitational Theory; Garth A Barber; arXiv:gr-cq / 0405094v3 10 June 2004.

[33] Essential Mathematical Methods for Physicists ; Weber and Arfken ; 2004.

[35] Existence and smoothness of the Navier Stokes equation; Charles F. Feffermann; May 1, 2000. Princeton University, Department of Mathematics, Princeton, NJ 08544-1000

[36] Intrinsic formulae for the Casimir operators of semi direct products of the exceptional Lie algebra G_2 and a Heisenberg Lie algebra; R. Campoamor-Stursberg; Laboratoires de Mathématiques et Applications, Université de Haute Alsace, Mulhouse; 18 Juin 2004.

[37] Rapport made by Professor Wesson for the California Institute for Physics and Astrophysics concerning future research around the ZPF.

Haut du document

II. The (E) question

1. Connecting with the actual language of mathematic

We will now make a difficult work of normalization of our own theory. That is we consider all preceding developments only as a spontaneous intuition leading to astonishing results and that our way of reasoning is showing a lack of generality. For example, we introduced the (E) question during the demonstration of the Maxwell's force and never more spoke again about it.

Nevertheless it is possible to connect what we call the elastic wedge product with some other well known mathematical construction.

To demonstrate this assertion we will refer to the work of R. Campoamor-Stursberg [36] (The notation of this author will be write in blue) and consider that our space E could become a Lie algebra $\breve{\mathbf{g}}$, our basis $\Omega(\mathbf{e}_1, \ldots, \mathbf{e}_N)$ could be the $\{X_1, \ldots, X_N\}$, our cube of the so-called projectors $\blacktriangledown A = \{A_k{}^i{}_j\}$ (1) could be the structure constants over this basis $\{C_i{}^k{}_j\}$ if we have $- C_i{}^k{}_j = A_j{}^i{}_k$ (2) and consequently the representation of $\breve{\mathbf{g}}$ in the space $C^{\propto}(\breve{\mathbf{g}}^*)$ by means of differential operators $X^*_i = - C_i{}^k{}_j .\ x^k .\ \partial/\partial x^j$ could be interpreted as the elastic wedge product of ∂ and $\mathbf{x}$, that is (with our notation) $\mathbf{X^*} = \nabla ⌂ \mathbf{x}$ (3) where ∇ is the vector operator gradient and where finally the brackets $[X_i, X_j] = C_i{}^k{}_j .\ X_k\ (1 \leq i < j \leq N)$ could be related to our trivial matrix. In this context, an analytic function $F \in C^{\propto}(\breve{\mathbf{g}}^*)$ is called an invariant of $\breve{\mathbf{g}}$ if and only if it is a solution of the system: $\{X^*_i\ F = 0, (1 \leq i \leq N)\}$ that we can resume in $\nabla F ⌂ \mathbf{x} = \mathbf{0}$ (4) with our own notation.

The (E) question would have been in this special case: what are the squared matrix [P] and the vectors **z** allowing the splitting $\nabla F ⌂ \mathbf{x} = [P].\ \mathbf{x} + \mathbf{z} = \mathbf{0}$? This clearly demonstrates that the (E) question is including, *per construction*, the question of the invariant analytic functions of an Lie algebra which appears to be a reduction of our problematic to a zero elastic wedge product between the gradient of any analytic function F and any vector **x**; and consequently even that of the Casimir operators which is the reduction of the latter problematic to polynomial functions.