ThPériat / section 09 / Main results of the systematic analyse of the (E) question in a 3D space / 21 December 2004

1.1. Splitting in any basis

The last section demonstrates clearly the necessity to be in possession of a good comprehension of the (E) analyze. We will resume here the main results of this approach. As explained in other parts of this small work, we are asking on the opportunity to write equations like $^{(N)}\mathbf{u} \,⌂\, ^{(N)}\mathbf{w} = [^{(N)}P].\, ^{(N)}\mathbf{w} + ^{(N)}\mathbf{z}$ for N = 3 and for N = 4, where N is the dimension of the mathematical space E within of which the discussions take place. For ordinary vectors (N = 3) and to simplify the notation, we will no more write the $^{(3)}$ before the letter symbolizing a vector. Vectors **u** and **w** have here no special physical signification. So all results exposed now are concerning a systematic analyze of the following relation, according to the intuition that any splitting could be related to the underground geometry of E :

$\mathbf{u} \,⌂\, \mathbf{w} = [P].\, \mathbf{w} + \mathbf{z}$ (1)

The first thing to remark concerning (1) is that it automatically implies the existence of a conoide f(**u**) which is in fact nothing else than the determinant of (1) written as a linear system depending on **w**.

$|S(w)| = f(\mathbf{u}) = c_{11}.\,(u^1)^2 + c_{22}.\,(u^2)^2 + c_{33}.\,(u^3)^2 + c_{12}.\,u^1.\,u^2 + c_{23}.\,u^2.\,u^3 + c_{13}.\,u^1.\,u^3 + c_1.\,u^1 + c_2.\,u^2 + c_3.\,u^3 - |P|$

[Noemie04. Pdf; page 27]

where the c_{ab} and the c_a are respectively the bilinear and the linear coefficients of this conoide. One can easily demonstrates that all of them are depending on the components of the matrix [A] defining the ewp on the basis Ω of E where one works and on the components of [P], the last (tenth) coefficient only being the determinant of this latter. The ten relations are the system ℘.

After complicated calculations it is possible to demonstrate that:

$T(.)(\Omega, \Omega).\,[R] = ½.\,[S_0] - (1/|A|).\,\Phi([S_0]^{-1}.\,\mathbf{c})$

[Noemie04. Pdf; page 25]

With:

- (i) **c** is the vector owning the linear coefficients c_a (with a = 1, 2 and 3) of the conoide f(**u**) as components on Ω.
- (ii) $[S_0] = T_2(o)(\partial_{\mathbf{u}}, \partial_{\mathbf{u}}(\mathbf{u}))$ [Noemie04.pdf ; page 29]
- (iii) $\mathbf{s} = -[S_0]^{-1}.\,\mathbf{c}$ is the singular vector of f(**u**) only existing when $|S_0| = \Delta \neq 0$. [Noemie04.pdf ; page 30]

under following crucial conditions:

$[T].\,[P] = T_2(.)(\Omega, \Omega).\,[R]$ [Noemie04. pdf; page 16; (50)]

$|A| \neq 0$ $|A| = \pm 1$

where:

- (i) $[T]^{-1} = |A|.\,[A]^t.\,[J]$ [Noemie04. pdf; page 17; (60)]
- (ii) $T_2(.)(\Omega, \Omega)$ is the representation of the metric tensor of E on Ω

with [A]:

$$\begin{matrix} A_{12}^{\ 1} & A_{12}^{\ 2} & A_{12}^{\ 3} \\ A_{23}^{\ 1} & A_{23}^{\ 2} & A_{23}^{\ 3} \\ A_{13}^{\ 1} & A_{13}^{\ 2} & A_{13}^{\ 3} \end{matrix}$$

And:

$$[J] = \begin{matrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{matrix}$$

All these things resulting at the end in:

$[P] = ½.\,|A|.\,\{[A]^t.\,[J]\}.\,T_2(o)(\partial_{\mathbf{u}}, \partial_{\mathbf{u}} f(\mathbf{u})) + \{[A]^t.\,[J]\}.\,\Phi(\mathbf{s})$

[Noemie04.pdf ; page 29 ; (172)]

1.2. trivial splitting

Trivial splitting are those where (1) is reduced in:

$\mathbf{u} \,⌂\, \mathbf{w} = [P].\, \mathbf{w}$ (1.0)

The simplest one is obtained without calculation and without any consideration regarding **w** in writing:

$[P] = \Phi(\mathbf{u})$ (2)

But the latter in not the only one and the condition that has to hold to be sure to own a trivial splitting [P] is:

$|[P] = \Phi(\mathbf{u})| = 0$ (3)

It is really easy to verify that if (2) holds, than (3) too.

In order to get a first consistent approach of this analyze, we first reduce it to basis where the elastic wedge product is the classic wedge product of two ordinary vectors in a right hand basis; this hypothesis is equivalent to:

$[A] = [J]$ (4)

$|A| = -1$ (5)

and logically:

$[P] = -½.\,T_2(o)(\partial_{\mathbf{u}}, \partial_{\mathbf{u}} f(\mathbf{u})) + \Phi(\mathbf{s})$ (6)

Since one can also demonstrate that:

$\partial_{\mathbf{u}} f(\mathbf{u}) = [S_0].\,(\mathbf{u} - \mathbf{s})$ [Noemie04.pdf ; page 30 ; (170)]

As soon as $\Delta \neq 0$ one gets:

$\mathbf{s} = -T^{-1}{}_2(o)(\partial_{\mathbf{u}}, \partial_{\mathbf{u}} f(\mathbf{u})).\,\partial_{\mathbf{u}} f(\mathbf{u}) + \mathbf{u}$ (7)

and consequently the condition (3) becomes:

$|-½.\,[S_0] - \Phi([S_0]^{-1}.\,\partial_{\mathbf{u}} f(\mathbf{u}))| = 0$ (8)

As we used to do, we introduce the Io vector:

$\mathbf{IO} = -T^{-1}{}_2(o)(\partial_{\mathbf{u}},\partial_{\mathbf{u}} f(\mathbf{u})).\,\partial_{\mathbf{u}} f(\mathbf{u})$ (9)

And we decide to calculate the determinant D such as:

$D = |-½.\,T_2(o)(\partial_{\mathbf{u}},\partial_{\mathbf{u}} f(\mathbf{u})) + \Phi(\mathbf{IO})|$ (10)

without consideration for the definition (9) and independently of the value of Δ with the intention to win the more general results. After long calculations we get:

$D = ⅛.\,|T_2(o)(\partial_{\mathbf{u}}, \partial_{\mathbf{u}} f(\mathbf{u}))|$
$+ ¼.\,[\partial_{11} f.\,(\partial_{32} f - \partial_{23} f) + (\partial_{13} f.\,\partial_{21} f - \partial_{12} f.\,\partial_{31} f)].\,(\mathbf{IO})^1$
$+ ¼.\,[\partial_{22} f.\,(\partial_{31} f - \partial_{13} f) + (\partial_{21} f.\,\partial_{32} f - \partial_{12} f.\,\partial_{23} f)].\,(\mathbf{IO})^2$
$+ ¼.\,[\partial_{33} f.\,(\partial_{21} f - \partial_{12} f) + (\partial_{13} f.\,\partial_{32} f - \partial_{31} f.\,\partial_{23} f)].\,(\mathbf{IO})^3$
$+ ½.\,(\partial_{12} f + \partial_{21} f).\,(\mathbf{IO})^1.\,(\mathbf{IO})^2$
$+ ½.\,(\partial_{32} f + \partial_{23} f).\,(\mathbf{IO})^2.\,(\mathbf{IO})^3$
$+ ½.\,(\partial_{13} f + \partial_{31} f).\,(\mathbf{IO})^1.\,(\mathbf{IO})^3$
$+ ½.\,\partial_{11} f.\,((\mathbf{IO})^1)^2$
$+ ½.\,\partial_{22} f.\,((\mathbf{IO})^2)^2$
$+ ½.\,\partial_{33} f.\,((\mathbf{IO})^3)^2$
(11)

This important result allows a "come back" to our ordinary conoide f(**u**) which obviously is satisfying a relation like:

$\partial_{km} f - \partial_{mk} f = 0$ (12)

if we consider that any splitting is occurring at each short instant of a chronology, thus allowing to write with a good approximation:

$\partial f(\mathbf{u})/\partial u^1 = 2.\,c_{11}.\,(u^1) + c_{12}.\,u^2 + c_{13}.\,u^3 + c_1$
$\partial f(\mathbf{u})/\partial u^2 = c_{12}.\,(u^1) + 2.\,c_{22}.\,u^2 + c_{23}.\,u^3 + c_2$
$\partial f(\mathbf{u})/\partial u^3 = c_{13}.\,(u^1) + c_{23}.\,u^2 + 2.\,c_{33}.\,u^3 + c_3$ (13)

and:

$\partial^2 f(\mathbf{u})/\,\partial u^1 \partial u^1 = 2.\,c_{11}$
$\partial^2 f(\mathbf{u})/\,\partial u^1 \partial u^2 = c_{12}$
$\partial^2 f(\mathbf{u})/\,\partial u^1 \partial u^3 = c_{13}$
$\partial^2 f(\mathbf{u})/\,\partial u^2 \partial u^1 = c_{12}$
$\partial^2 f(\mathbf{u})/\,\partial u^2 \partial u^2 = 2.\,c_{22}$
$\partial^2 f(\mathbf{u})/\,\partial u^2 \partial u^3 = c_{23}$
$\partial^2 f(\mathbf{u})/\,\partial u^3 \partial u^1 = c_{13}$
$\partial^2 f(\mathbf{u})/\,\partial u^3 \partial u^2 = c_{23}$
$\partial^2 f(\mathbf{u})/\,\partial u^3 \partial u^3 = 2.\,c_{33}$ (14)

That is in fact:

$[S_0] = T_2(o)(\partial_{\mathbf{u}}, \partial_{\mathbf{u}}(\mathbf{u}))$ [Noemie04.pdf ; page 29 ; (171)]

According to this hypothesis (12) the determinant D is reduced in:

$D = ⅛. \Delta + ½. \sum_a \sum_b \partial_{ab}f. (\mathbf{IO})^a. (\mathbf{IO})^b$ (15)

To finish the calculations giving at the end the trivial splitting, we should now take care of the definition (9) that can also be written:

$(\mathbf{IO})^a = - \sum_k S_{a\,k\cdot} (\partial f(\mathbf{u})/\partial u^k) = (- [S_0]^{-1}. \partial_{\mathbf{u}} f(\mathbf{u}))^a$ (16)

where the $S_{a\,k\cdot}$ are the components of $[S_0]^{-1}$.

All trivial splitting **u** ∧ **w** = [P]. **w** are characterized by (8), that is here by:

$D = 0$ (17)

That is, more precisely, because of (15):

$0 = \Delta + 4. \sum_a \sum_b \partial_{ab}f. (\mathbf{IO})^a. (\mathbf{IO})^b$ (18)

And because of (16) one obtains a relation like:

$0 = ⅛. \Delta + ½. \sum_k \sum_m (\sum_a \sum_b \partial_{ab}f. S_{a\,k\cdot} S_{b\,m}). \partial_k f. \partial_m f$ (19)

Where:

$\partial f(\mathbf{u})/\partial u^k = \partial_k f$ (18)

We will see more results in the next section.