

1.3. Determinant of [P]

To calculate the determinant of the matrices [P] obtained with any splitting, we first make use of:

$$T(\cdot)(\Omega, \Omega). [R] = \frac{1}{2}. [S_0] - (1/|A|). \Phi([S_0]^{-1}. \mathbf{c})$$

[\[Noemie04.Pdf; page 25; \(160\)\]](#)

And calculate the determinant of $T(\cdot)(\Omega, \Omega). [R]$ that we will note: $g. |R|$ where evidently g is the determinant of $T(\cdot)(\Omega, \Omega)$ and $|R|$ the determinant of $[R]$. The formalism of $T(\cdot)(\Omega, \Omega). [R]$ implies the calculation of:

$$\begin{vmatrix} c_{11} & (c_{12}/2) + \varepsilon.c_j. S_{3j} & (c_{13}/2) - \varepsilon.c_j. S_{2j} \\ (c_{12}/2) - \varepsilon.c_j. S_{3j} & c_{22} & (c_{23}/2) + \varepsilon.c_j. S_{1j} \\ (c_{13}/2) + \varepsilon.c_j. S_{2j} & (c_{23}/2) - \varepsilon.c_j. S_{1j} & c_{33} \end{vmatrix} \quad (1)$$

Where the convention concerning the definition of the ewp on the basis Ω (left hand or right hand) is given through:

$$\varepsilon = (1/|A|) = \pm 1 \quad (2)$$

This leads to:

$$g. |R| = \frac{1}{8}. \Delta + \sum_a \sum_b (c_{ab}). \sum_j [c_j. S_{aj}]. \sum_k [c_k. S_{bk}] \quad (3)$$

independently of the convention concerning the definition of the ewp on the basis Ω (left hand or right hand).

As long as the following crucial conditions hold:

$$[T]. [P] = T_2(\cdot)(\Omega, \Omega). [R] \quad \text{[Noemie04.pdf; page 16; (50)]}$$

$$|A| \neq 0 \quad |A| = \pm 1$$

where:

(i) $[T]^{-1} = |A|. [A]^t. [J]$ [\[Noemie04.pdf; page 17; \(60\)\]](#)

(ii) $T_2(\cdot)(\Omega, \Omega)$ is the representation of the metric tensor of E on Ω

This implies:

$$|P| = -g. |R| \quad (4)$$

This consequently means that we have to pay attention to the quantity:

$$T = \sum_a \sum_b (c_{ab}). \sum_j [c_j. S_{aj}]. \sum_k [c_k. S_{bk}] \quad (5)$$

Accordingly to the fact that we could demonstrate that:

(i) the matrix $T(\cdot)(\Omega, \Omega). [R]$ must have the following formalism:

$$\begin{vmatrix} c_{11} & gr_{12} & gr_{13} \\ (c_{12} - gr_{12}) & c_{22} & gr_{23} \\ (c_{13} - gr_{13}) & (c_{23} - gr_{23}) & c_{33} \end{vmatrix} \quad (6)$$

(ii) following relations hold:

$$\Delta. gr_{23} = (c_1/|A|). \Delta. S_{11} + (c_2/|A|). \Delta. S_{12} + (c_3/|A|). \Delta. S_{13} + c_{23}. \Delta/2 \quad (7)$$

$$\Delta. gr_{13} = -(c_1/|A|). \Delta. S_{21} - (c_2/|A|). \Delta. S_{22} - (c_3/|A|). \Delta. S_{23} + c_{13}. \Delta/2 \quad (8)$$

$$\Delta. gr_{12} = (c_1/|A|). \Delta. S_{31} + (c_2/|A|). \Delta. S_{32} + (c_3/|A|). \Delta. S_{33} + c_{12}. \Delta/2 \quad (9)$$

It is easy to get:

$$|A|. \Delta. [gr_{23} - c_{23}/2] = c_1. \Delta. S_{11} + c_2. \Delta. S_{12} + c_3. \Delta. S_{13} \quad (10)$$

$$|A|. \Delta. [gr_{13} - c_{13}/2] = -c_1. \Delta. S_{21} - c_2. \Delta. S_{22} - c_3. \Delta. S_{23} \quad (11)$$

$$|A|. \Delta. [gr_{12} - c_{12}/2] = c_1. \Delta. S_{31} + c_2. \Delta. S_{32} + c_3. \Delta. S_{33} \quad (12)$$

Introducing here a vector \mathbf{T} with ordinary coordinates:

$$+ |A|. \Delta. [gr_{23} - c_{23}/2] = T^1 \quad (13)$$

$$- |A|. \Delta. [gr_{13} - c_{13}/2] = T^2 \quad (14)$$

$$+ |A|. \Delta. [gr_{12} - c_{12}/2] = T^3 \quad (15)$$

one rapidly see that (10), (11) and (12) can be resumed with:

$$\mathbf{T} = \Delta. [S_0]^{-1}. \mathbf{c} \quad (16)$$

Where obviously the gr_{ij} are the components of a matrix $[S_0]^{-1}$.

Thus relation (5) can be transformed in:

$$\Delta^2. T = \sum_a \sum_b (c_{ab}). \sum_j [c_j. \Delta. S_{aj}]. \sum_k [c_k. \Delta. S_{bk}] \quad (17)$$

or, with (16), in:

$$\Delta^2. T = \sum_a \sum_b c_{ab}. T^a. T^b \quad (18)$$

From (4), it follows :

$$|P| = -g. |R| = -\frac{1}{8}. \Delta - T$$

And at the end:

$$\frac{1}{8}. \Delta^3 - \Delta^2. |P| + \sum_a \sum_b c_{ab}. T^a. T^b = 0 \quad (19)$$

Since we know that [\[Noemie04.pdf; page 30\]](#):

$$\Delta \neq 0 \Leftrightarrow \exists \mathbf{s} = -[S_0]^{-1}. \mathbf{c} \quad (20)$$

The relation (16) can be written:

$$\mathbf{T} = -\Delta. \mathbf{s} \quad (21)$$

So that relation (19) gives:

$$\Delta \neq 0 \Leftrightarrow \sum_a \sum_b c_{ab}. (T^b/\Delta). (T^b/\Delta) + \frac{1}{8}. \Delta - |P| = 0 \quad (22)$$

leading because of (21) to:

$$\Delta \neq 0 \Leftrightarrow \sum_a \sum_b c_{ab}. s^a. s^b + \frac{1}{8}. \Delta - |P| = 0 \quad (23)$$

1.4. Coherence

Since we know that (see section 09 § 1.2.):

$$\mathbf{s} - \mathbf{u} = \mathbf{0} \quad (1)$$

A comparison between [\[section 09 § 1.2. \(15\)\]](#) and [\[section 10 § 1.3. \(23\)\]](#) leads to:

$$D - \frac{1}{2}. \sum_a \sum_b \partial_{ab}f. (\mathbf{s} - \mathbf{u})^a. (\mathbf{s} - \mathbf{u})^b = |P| - \sum_a \sum_b c_{ab}. s^a. s^b \quad (2)$$

From this relation and because of [\[section 09 § 1.2. \(4\)\]](#) and of [\[section 09 § 1.2. \(5\)\]](#) we can deduce an important equation concerning *all trivial splitting* $\mathbf{u} \wedge \mathbf{w} = [P]$. \mathbf{w} which are characterized by [\[section 09 § 1.2. \(8\)\]](#), that is here by:

$$D = 0 \quad (3)$$

Or in extenso:

$$-\frac{1}{2}. \sum_a \sum_b \partial_{ab}f. (\mathbf{s} - \mathbf{u})^a. (\mathbf{s} - \mathbf{u})^b = |P| - \sum_a \sum_b c_{ab}. s^a. s^b \quad (4)$$

For all products of *any vector u with itself* (thus studying the $\mathbf{u} \wedge \mathbf{u} = [P]$. $\mathbf{u} = \mathbf{0}$ configurations implying $|P| = 0$), this is still reducing in:

$$\frac{1}{2}. \sum_a \sum_b \partial_{ab}f. (\mathbf{s} - \mathbf{u})^a. (\mathbf{s} - \mathbf{u})^b = \sum_a \sum_b c_{ab}. s^a. s^b \quad (5)$$

This result can be written in making use of the set [\[section 09 § 1.2. \(14\)\]](#) to get a coherent notation; thus:

$$\frac{1}{2}. \sum_a \partial_{aa}f. (s^a - u^a)^2 + \frac{1}{2}. \sum_a \sum_{b \neq a} \partial_{ab}f. (s^a - u^a). (s^b - u^b) = \sum_a c_{aa}. (s^a)^2 + \sum_a \sum_{b \neq a} c_{ab}. s^a. s^b$$

$$\frac{1}{2}. \sum_a \partial_{aa}f. (s^a - u^a)^2 + \frac{1}{2}. \sum_a \sum_{b \neq a} \partial_{ab}f. (s^a - u^a). (s^b - u^b) =$$

$$\frac{1}{2}. \sum_a \partial_{aa}f. (s^a)^2 + \sum_a \sum_{b \neq a} \partial_{ab}f. s^a. s^b$$

$$\frac{1}{2}. \sum_a \partial_{aa}f. (s^a)^2 + \frac{1}{2}. \sum_a \partial_{aa}f. (u^a)^2 - \sum_a \partial_{aa}f. s^a. u^a + \frac{1}{2}. \sum_a \sum_{b \neq a} \partial_{ab}f. s^a. s^b - \frac{1}{2}. \sum_a \sum_{b \neq a} \partial_{ab}f. (s^a. u^b) - \frac{1}{2}. \sum_a \sum_{b \neq a} \partial_{ab}f. u^a. s^b + \frac{1}{2}. \sum_a \sum_{b \neq a} \partial_{ab}f. u^a. u^b =$$

$$\frac{1}{2}. \sum_a \partial_{aa}f. (s^a)^2 + \sum_a \sum_{b \neq a} \partial_{ab}f. s^a. s^b$$

$$\frac{1}{2}. \sum_a \partial_{aa}f. (u^a)^2 + \frac{1}{2}. \sum_a \sum_{b \neq a} \partial_{ab}f. u^a. u^b - \sum_a \partial_{aa}f. s^a. u^a - \frac{1}{2}. \sum_a \sum_{b \neq a} \partial_{ab}f. s^a. u^b$$

$$- \frac{1}{2}. \sum_a \sum_{b \neq a} \partial_{ab}f. u^a. s^b$$

$$- \frac{1}{2}. \sum_a \sum_{b \neq a} \partial_{ab}f. s^a. s^b$$

$$= 0 \quad (6)$$

This is a conoide and one could expect for the coherence of all calculations that we have made until now that it would be $f(\mathbf{u})$. Next paragraphs are the demonstration of this coherence.

A. The bilinear part of $f(\mathbf{u})$:

Once more time with the help of [section 09 § 1.2. (14)] it is extremely easy to control that terms one and two above are:

$$\sum_a c_{aa} \cdot (u^a)^2 + \frac{1}{2} \cdot \sum_a \sum_{b \neq a} c_{ab} \cdot u^a \cdot u^b$$

The second term contains c_{ab} coefficients with a subscript a greater than b that are not appearing in the original formulation of $f(\mathbf{u})$.

But if one consider that per principle:

$$c_{ab} = c_{ba} \quad (7)$$

the problem concerning the second term, containing so many 2-forms with a subscript a greater than b as 2-forms with a subscript a smaller than b (all forms appearing in a symmetric way) disappears and one get:

$$\sum_a c_{aa} \cdot (u^a)^2 + \sum_{a < b} \sum_{b \neq a} c_{ab} \cdot u^a \cdot u^b$$

That is in fact the bilinear part of $f(\mathbf{u})$.

B. The tenth component

Because we do have in this special case of a product of a vector \mathbf{u} with itself the obligatory condition $|P| = 0$ (8) to get the insurance that our result will not be depending on the choice that we will do for this vector \mathbf{u} , the constant term of $f(\mathbf{u})$ should not be present; indeed, observing (6) in all details shows no constant term.

C. The linear part of $f(\mathbf{u})$

Thus:

$$-\sum_a (\partial_{aa} f \cdot s^a \cdot u^a + \frac{1}{2} \cdot \sum_{b \neq a} \partial_{ab} f \cdot s^a \cdot u^b + \frac{1}{2} \cdot \sum_{b \neq a} \partial_{ab} f \cdot u^a \cdot s^b)$$

$$= -\sum_a [\partial_{aa} f \cdot s^a \cdot u^a + \frac{1}{2} \cdot \sum_{b \neq a} (\partial_{ab} f + \partial_{ba} f) \cdot u^a \cdot s^b]$$

$$= -\sum_a [\partial_{aa} f \cdot s^a + \frac{1}{2} \cdot \sum_{b \neq a} (\partial_{ab} f + \partial_{ba} f) \cdot s^b] \cdot u^a \quad (9)$$

should be now identify with:

$$c_1 \cdot u^1 + c_2 \cdot u^2 + c_3 \cdot u^3 = \sum_a c_a \cdot u^a = \sum_a u^a \cdot c_a \quad (10)$$

This would lead to:

$$c_a = -[\partial_{aa} f \cdot s^a + \frac{1}{2} \cdot \sum_{b \neq a} (\partial_{ab} f + \partial_{ba} f) \cdot s^b] \quad (11)$$

Making use of [section 09 § 1.2. (14)] is leading to:

$$c_a = -[2 \cdot c_{aa} \cdot s^a + \sum_{b \neq a < c} c_{ab} \cdot s^b] \quad (12)$$

We recognize the inverse relation of (20):

$$\mathbf{c} = -[S_0] \cdot \mathbf{s} \quad (13)$$

and we now have the total insurance that our calculations were correct. With the help of A. B. and C. above we demonstrated that (5) is at the end equivalent to:

$$f(\mathbf{u}) = 0 \quad (14)$$

for the $\mathbf{u} \wedge \mathbf{u} = [P]$. $\mathbf{u} = \mathbf{0}$ configurations.

1.5. Interpretation

As we explained in § 1.1. , writing $\mathbf{u} \triangle \mathbf{w} = [P]$. $\mathbf{w} + \mathbf{z}$ is automatically implying the existence of a conoide $f(\mathbf{u})$. Coefficients of the conoide are satisfying a system \wp of 10 equations:

$$c_{11} = p_{11} \cdot (A_{12}^3 \cdot A_{13}^2 - A_{12}^2 \cdot A_{13}^3) + p_{21} \cdot (A_{12}^1 \cdot A_{13}^3 - A_{12}^3 \cdot A_{13}^1) + p_{31} \cdot (A_{12}^2 \cdot A_{13}^1 - A_{12}^1 \cdot A_{13}^2) \quad (15-1)$$

$$c_{22} = p_{12} \cdot (A_{12}^3 \cdot A_{23}^2 - A_{12}^2 \cdot A_{23}^3) + p_{22} \cdot (A_{12}^1 \cdot A_{23}^3 - A_{12}^3 \cdot A_{23}^1) + p_{32} \cdot (A_{12}^2 \cdot A_{23}^1 - A_{12}^1 \cdot A_{23}^2) \quad (15-2)$$

$$c_{33} = p_{13} \cdot (A_{13}^3 \cdot A_{23}^2 - A_{13}^2 \cdot A_{23}^3) + p_{23} \cdot (A_{13}^1 \cdot A_{23}^3 - A_{13}^3 \cdot A_{23}^1) + p_{33} \cdot (A_{13}^2 \cdot A_{23}^1 - A_{13}^1 \cdot A_{23}^2) \quad (15-3)$$

$$c_{12} = p_{11} \cdot (A_{12}^3 \cdot A_{23}^2 - A_{12}^2 \cdot A_{23}^3) + p_{12} \cdot (A_{13}^1 \cdot A_{12}^3 - A_{13}^3 \cdot A_{12}^1) + p_{21} \cdot (A_{12}^1 \cdot A_{23}^3 - A_{12}^3 \cdot A_{23}^1) + p_{22} \cdot (A_{13}^3 \cdot A_{12}^1 - A_{13}^1 \cdot A_{12}^3) + p_{31} \cdot (A_{12}^2 \cdot A_{23}^2 - A_{12}^2 \cdot A_{23}^2) + p_{32} \cdot (A_{13}^1 \cdot A_{12}^2 - A_{13}^2 \cdot A_{12}^1) \quad (15-4)$$

$$c_{13} = p_{11} \cdot (A_{13}^2 \cdot A_{23}^3 - A_{13}^3 \cdot A_{23}^2) + p_{13} \cdot (A_{12}^3 \cdot A_{13}^1 - A_{12}^1 \cdot A_{13}^3) + p_{21} \cdot (A_{13}^1 \cdot A_{23}^3 - A_{13}^3 \cdot A_{23}^1) + p_{23} \cdot (A_{12}^1 \cdot A_{13}^3 - A_{12}^3 \cdot A_{13}^1) + p_{31} \cdot (A_{13}^2 \cdot A_{23}^3 - A_{13}^3 \cdot A_{23}^2) + p_{33} \cdot (A_{12}^2 \cdot A_{13}^3 - A_{12}^3 \cdot A_{13}^2) \quad (15-5)$$

$$c_{23} = p_{12} \cdot (A_{23}^2 \cdot A_{13}^3 - A_{23}^3 \cdot A_{13}^2) + p_{13} \cdot (A_{12}^3 \cdot A_{23}^1 - A_{12}^1 \cdot A_{23}^3) + p_{22} \cdot (A_{23}^1 \cdot A_{13}^3 - A_{23}^3 \cdot A_{13}^1) + p_{23} \cdot (A_{12}^1 \cdot A_{23}^3 - A_{12}^3 \cdot A_{23}^1) + p_{32} \cdot (A_{23}^2 \cdot A_{13}^3 - A_{23}^3 \cdot A_{13}^2) + p_{33} \cdot (A_{12}^2 \cdot A_{23}^3 - A_{12}^3 \cdot A_{23}^2) \quad (15-6)$$

$$c_1 = A_{12}^1 \cdot (p_{31} \cdot p_{23} - p_{21} \cdot p_{33}) + A_{12}^2 \cdot (p_{33} \cdot p_{11} - p_{31} \cdot p_{13}) + A_{12}^3 \cdot (p_{21} \cdot p_{13} - p_{11} \cdot p_{23}) + A_{13}^1 \cdot (p_{21} \cdot p_{32} - p_{22} \cdot p_{31}) + A_{13}^2 \cdot (p_{31} \cdot p_{12} - p_{11} \cdot p_{32}) + A_{13}^3 \cdot (p_{11} \cdot p_{22} - p_{21} \cdot p_{12}) \quad (15-7)$$

$$c_2 = A_{12}^1 \cdot (p_{32} \cdot p_{23} - p_{22} \cdot p_{33}) + A_{12}^2 \cdot (p_{12} \cdot p_{33} - p_{32} \cdot p_{13}) + A_{12}^3 \cdot (p_{13} \cdot p_{22} - p_{12} \cdot p_{23}) + A_{23}^1 \cdot (p_{21} \cdot p_{32} - p_{31} \cdot p_{22}) + A_{23}^2 \cdot (p_{31} \cdot p_{12} - p_{32} \cdot p_{11}) + A_{23}^3 \cdot (p_{22} \cdot p_{11} - p_{21} \cdot p_{12}) \quad (15-8)$$

$$c_3 = A_{13}^1 \cdot (p_{32} \cdot p_{23} - p_{22} \cdot p_{33}) + A_{13}^2 \cdot (p_{12} \cdot p_{33} - p_{32} \cdot p_{13}) + A_{13}^3 \cdot (p_{13} \cdot p_{22} - p_{12} \cdot p_{23}) + A_{23}^1 \cdot (p_{21} \cdot p_{33} - p_{31} \cdot p_{23}) + A_{23}^2 \cdot (p_{31} \cdot p_{13} - p_{33} \cdot p_{11}) + A_{23}^3 \cdot (p_{23} \cdot p_{11} - p_{21} \cdot p_{13}) \quad (15-9)$$

$$\mathbf{c} = -|P| \quad (15-10)$$

From this formulation one can easily guess that $f(\mathbf{u})$ could be a stable conoide in a changing environment (a variable definition of the ewp) if the components of $[P]$ would change accordingly to the variations of ${}^{(3)}\nabla A$.

Otherwise, if ${}^{(3)}\nabla A$ is invariant (which is for example the case in a right hand basis where the ewp is the classic wedge product), we saw that several matrices can be associated to a unique conoide (e. g. all trivial splitting matrices $[P]$ for the $\mathbf{u} \wedge \mathbf{u} = [P]$. $\mathbf{u} = \mathbf{0}$ configurations are associated with a $f(\mathbf{u}) = 0 \forall \mathbf{u}$).

But in general, we can reasonably expect that a set of conoide can be associated to the vector \mathbf{u} undergoing the splitting action of \mathbf{w} .

It's quite easy to see that [section 09; § 1.2. (13)] can be written:

$$\partial_{\mathbf{u}} f(\mathbf{u}) = [S_0] \cdot \mathbf{u} + \mathbf{c} \quad (16)$$

during we know that the singular vector \mathbf{s} is such that:

$$\partial_{\mathbf{u}} f(\mathbf{s}) = \mathbf{0} = [S_0] \cdot \mathbf{s} + \mathbf{c} \quad (17)$$

From this we deduce:

$$\partial_{\mathbf{u}} f(\mathbf{u}) = [S_0] \cdot (\mathbf{u} - \mathbf{s}) \quad (18)$$

So that in the set of conoides associated to \mathbf{u} , those which are characterized by $\partial_{\mathbf{u}} f(\mathbf{u}) = \mathbf{0}$ (19) determinate a special family. Three possibilities arise from this situation described by:

$$\{f \in \mathfrak{T}(E; K) \mid \partial_{\mathbf{u}} f(\mathbf{u}) = \mathbf{0} \Rightarrow [S_0]. (\mathbf{u} - \mathbf{s}) = \mathbf{0}\} \quad (20)$$

- (i) $[S_0] = [0]$ (21) $\forall \mathbf{IO}$; in this situation $\Delta = 0$ and the $[S_0]$ matrix is not invertible. There is no singular vector. The function $f(\mathbf{u})$ is reduced to $|S(\mathbf{w})| = f(\mathbf{u}) = c_1 \cdot u^1 + c_2 \cdot u^2 + c_3 \cdot u^3 - |P|$ (22)
- (ii) $[S_0] = 0$ (23) $\forall \mathbf{IO}$; in this situation $\Delta = 0$ and the $[S_0]$ matrix is not invertible. There is no singular vector but the function $f(\mathbf{u})$ is not reduced to (22).
- (iii) $\forall [S_0], \mathbf{IO} = (\mathbf{u} - \mathbf{s}) = \mathbf{0}$ (24). This third opportunity is really interesting because leading to $[P] = \frac{1}{2} \cdot |A| \cdot \{[A]^t, [J]\} \cdot T_2(o)(\partial_{\mathbf{u}}, \partial_{\mathbf{u}} f(\mathbf{u})) + \{[A]^t, [J]\} \cdot \Phi(\mathbf{s}) = \{[A]^t, [J]\} \cdot \Phi(\mathbf{u})$ in any basis and more particularly to $[P] = \Phi(\mathbf{u})$ (25) in a right hand basis where the ewp is the clasiic wedge product of two vectors. Thus we must make the important remark that the easiest trivial splitting, in extenso the trivial matrix $\Phi(\mathbf{u})$, is corresponding to special situations corresponding to: $\{\partial_{\mathbf{u}} f(\mathbf{u}) = \mathbf{0}, \forall [S_0]$ (not necessary equal to $[0]$) and $\mathbf{u} = \mathbf{s}\}$ (26).

The last (but not the least) consequence of this third result is that for all products of *any vector u with itself*, relation (5) is reducing in:

$$0 = \sum_a \sum_b c_{ab} \cdot u^a \cdot u^b \quad (27)$$

The $\mathbf{u} \wedge \mathbf{u} = [P]. \mathbf{u} = \mathbf{0}$ configurations can theoretically be described by $\{f(\mathbf{u}) = 0, \partial_{\mathbf{u}} f(\mathbf{u}) = \mathbf{0}, \forall [S_0]$ (not necessary equal to $[0]$), $\mathbf{u} = \mathbf{s}, |P| = 0$ and $0 = \sum_a \sum_b c_{ab} \cdot u^a \cdot u^b\}$. The “ $\forall [S_0]$ (not necessary equal to $[0]$)” must be explained. In fact if $\mathbf{IO} = (\mathbf{u} - \mathbf{s}) = \mathbf{0}$, and if $\Delta \neq 0$ then we can calculate D as made in [section 09; § 1.2] and we get $D = \frac{1}{8} \cdot \Delta$. But in this case $D \neq 0$ and this means that we have no trivial splitting; this case must be reject. The only compatible possibility to “reach” a trivial splitting with our way of thinking is to have $D = 0$. Thus the $\mathbf{u} \wedge \mathbf{u} = [P]. \mathbf{u} = \mathbf{0}$ configurations are equivalent to:

$$\{f(\mathbf{u}) = 0, \partial_{\mathbf{u}} f(\mathbf{u}) = \mathbf{0}, \forall [S_0]$$
 (not necessary equal to $[0]$) $\mid \Delta = 0, \mathbf{u} = \mathbf{s}, |P| = 0$ and $0 = \sum_a \sum_b c_{ab} \cdot u^a \cdot u^b\}$ (28)

This means: the $\partial_{ab} f$ are not necessary all equal to zero but they are not independent (this is resulting in 27 and in $\Delta = 0$).

If \mathbf{u} is a speed vector, then we interpret (27) as the equation of the trajectory of the physical phenomenon with speed \mathbf{u} in the basis Ω of E. The $[C] = [c_{ab}]$ matrix is the “metric tensor” for this phenomenon. Relation (7) gives the insurance that it is a symmetric matrix.

Normally, the nature always offers a local metric tensor at every place. Thus we could always find a “natural” matrix $[C]$. The components of the $[C]$ matrix are connected to the components of $[S_0]$ via the [section 09 § 1.2. (14)]. Thus we could always find a “natural” matrix $[P]$ via [section 09 § 1.2. (6)] to split any product $\mathbf{u} \wedge \mathbf{u} = [P]. \mathbf{u} = \mathbf{0}$. If the metric is locally flat ($[C] = I_3$) then (27) implies $\mathbf{u}^2 = 0$ and $\mathbf{u} = \mathbf{0}$.

1.6. Provisory conclusion

It appears to be a very interesting approach if we can extend it to a 4D space (eventually curved) and if we can apply it to the description of photons (for example) following geodesics.

We must recall that we could prove under certain conditions that $[F]$, the EM strength tensor in a 4D space, can be written $T(\cdot)({}^{(4)}\Omega_0, {}^{(4)}\Omega_0) \cdot \Phi({}^{(4)}\mathbf{X})$ where ${}^{(4)}\Omega_0$ is a Lorentz basis.

We will demonstrate later that \mathbf{IO} can be interpret as a spin and consequently, we propose to understand the [section 09 § 1.2. (15)] as an expression of the hamiltonien related to an interaction of \mathbf{IO} with itself, it means with a spin – spin interaction.

If we get success in this extension of our first approach, we certainly will win enlightening informations concerning a very modern preoccupation, i. e. the reality of the universalism of the free fall principle. It is actually strongly and often discussed (see [30], [39], [40]).

Bibliography

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