Contribution to the Yang-Mills Theory / Part 01

Abstract:

We should like to present the Yang-Mills theory from a new point of view. There is at the beginning of our approach no good reason to justify it. But we shall see that it leads to the demonstration of the Lorentz-Einstein Law.

0. Context:

Let us:

1°) work on a space vector (E_4, K) of dimension 4, built on K and referred to a canonical basis $\Omega = (\mathbf{e}_0, \ldots, \mathbf{e}_\gamma, \ldots, \mathbf{e}_3)$.

2°) start with the usual field-strength formulation in the Yang-Mills theory [01; page 18]:

(0.1.)

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + g.\,[A_\alpha, A_\beta]$$

where g is the Yang-Mills coupling constant. The brackets are defined by:

(0.2.)

$$F^{(i)}{}_{\alpha\beta} = \partial_\alpha A^{(i)}{}_\beta - \partial_\beta A^{(i)}{}_\alpha + g.\, f^{(i)}{}_{jk}.\, A^{(j)}{}_\alpha.\, A^{(k)}{}_\beta \text{ for } (i) = 1, 2, \ldots$$

where the $A^{(i)}{}_\alpha$ for $\alpha = 0, 1, 2$ and 3 and for (i) = 1, 2, … are the gauge fields and where the $f^{(i)}{}_{jk}$ denotes the structure constants of the Lie algebra under consideration. The Lie algebra of this discussion is built on a gauge group which we take to be a compact Lie group.

3°) define an inner product on (E_4, K) that we shall call an extended vector product so that:

(0.3.)

$$(E_4, K) \times (E_4, K) \rightarrow (E_4, K) \mid \forall\ \blacktriangledown A \in K_{444},\ \forall\ (\mathbf{u}, \mathbf{w}) \rightarrow \text{⌂}_{(\blacktriangledown A)}(\mathbf{u}, \mathbf{w}) = A_{\alpha\beta}{}^{\gamma}.\, u^\alpha.\, w^\beta.\, \mathbf{e}_\gamma$$

1. Reduction of the Faraday Maxwell tensor.

The purpose of this first section is to find a reduction for the Maxwell Faraday tensor (henceforth the tensor) within an approach involving the existence of a so-called extended vector product (evp). The initial proposition involves a trivial matrix Φ supposed to be given via the following relation:

(1.1.)

$$^{(4)}\mathbf{u}\ \text{⌂}_{(\blacktriangledown A)}\ {}^{(4)}\mathbf{w} = \Phi.\ {}^{(4)}\mathbf{w}$$

Proposition 1.1.:

Our first hypothesis consists to believe that we can associate this matrix with a representation of the tensor, in extenso:

(1.2.)

Maxwell Faraday tensor

=

scalar one. (metric tensor time trivial matrix) + scalar two. (transposed of the trivial matrix time metric tensor)

Corresponding to the following relation in a local matrix language:

(1.3.)

$$F = s_1.\, G.\, \Phi + s_2.\, \Phi^t.\, G$$

The relation (1.1.) can itself be written in a local coordinates language referred to the basis Ω. Accordingly to the definition of any extended vector product:

(1.4.)

$$A_{\alpha\beta}{}^{\gamma}.\, u^\alpha.\, w^\beta.\, \mathbf{e}_\gamma = \Phi_{\gamma\beta}.\, w^\beta.\, \mathbf{e}_\gamma$$

And exactly because Ω is a basis, this is yielding:

(1.5.)

$$(A_{\alpha\beta}{}^{\gamma}.\, u^\alpha - \Phi_{\gamma\beta}).\, w^\beta = 0$$

So that a representation of the trivial matrix Φ on Ω that will not be depending on the second 4-vector involved in (1.1.) is given by:

(1.6.)

$$A_{\alpha\beta}{}^{\gamma}.\, u^\alpha - \Phi_{\gamma\beta} = 0$$

The component in position (γ, β) of the transposed of this matrix is:
(1.7.)

$$(\Phi^t)_{\gamma\beta} = \Phi_{\beta\gamma}$$

Our proposition is now corresponding to:
(1.8.)

$$F_{\alpha\beta} = s_1.\ g_{\alpha\gamma}.\ \Phi_{\gamma\beta} + s_2.\ \Phi^t_{\alpha\gamma}.\ g_{\gamma\beta}$$

Making use of (1.6.) and (1.7):
(1.9)

$$F_{\alpha\beta} = (s_1.\ g_{\alpha\gamma}.\ A_{\varepsilon\beta}{}^{\gamma}).\ u^{\varepsilon} + s_2.\ \Phi_{\gamma\alpha}.\ g_{\gamma\beta}$$

At the end, the proposition for a representation of the tensor is:
(1.10)

$$F_{\alpha\beta} = (s_1.\ g_{\alpha\gamma}.\ A_{\varepsilon\beta}{}^{\gamma} + s_2.\ A_{\varepsilon\alpha}{}^{\gamma}.\ g_{\gamma\beta}).\ u^{\varepsilon}$$

Proposition 1.2.: A scenario to connect with the historical formulation:

1.2.1. Fundamental hypothesis :

If we consider that EM physical phenomenon are occurring when following conditions are realized:

a) The local cube defining the extended vector product supposed to be involved in the discussion contains 64 scalars (elements of K) corresponding *a priori* to any local connection;
b) The fundamental extended vector product under consideration is in fact the extended vector product of the "EM-potential 4-vector", i.e. $^{(4)}\mathbf{A}$, by the local position 4-vector, i.e. $^{(4)}\mathbf{x}$;
c) The "EM-potential 4-vector" is parallel transported with respect to the local position 4-vector.

1.2.2. Calculation:

Then:
1) starting from the (in fact reduced) historical definition of the tensor (involving neither the total derivates D nor complementary terms, e.g. Yang Mills, but only the partial derivation that I shall note here ∂):
(1.2.2.1.)

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

2) and supposing that usual universal rules of the differential calculus are locally valid, it is straightforward to demonstrate that hypothesis a), b) and c) above lead to:
(1.2.2.2.)

$$F_{\alpha\beta} = (g_{\alpha\gamma}.\ A_{\varepsilon\beta}{}^{\gamma} - A_{\varepsilon\alpha}{}^{\gamma}.\ g_{\gamma\beta}).\ A^{\varepsilon} + (\partial_\alpha g_{\varepsilon\beta} - \partial_\beta g_{\varepsilon\alpha}).\ A^{\varepsilon}$$

And it is effectively easy to recognize the equation (1.11) for $s_1 = - s_2 = 1$ in the first part of the right hand term of (1.2.2.2.) if the $u^{\varepsilon} = A^{\varepsilon}$ are now the contra-variant components of the EM potential four vector $^{(4)}\mathbf{A}$.

1.2.3. Conclusion:

Assuming the hypothesis a) b) and c) above allow to write in matrix language on Ω:
(1.2.3.1.)

$$F = G.\ \Phi - \Phi^t.\ G + [\ldots(\partial_\alpha g_{\varepsilon\beta} - \partial_\beta g_{\varepsilon\alpha}).\ A^{\varepsilon}\ldots]$$

where G is the matrix representation of the local metric tensor whilst Φ is those of the trivial matrix and Φ^t of its transposed.

Proposition 1.3.: When the connection is metric compatible:

1.3.1. Characterization:

For the coherence of this approach we shall now suppose that the cube A is coinciding with a metric compatible connection (Levi-Civita); this means that we can write the well-known formula:
(1.3.1.1.)

$$A_{\varepsilon\alpha}{}^{\gamma} = ½.\ g^{\gamma\varphi}.\ (\partial_\varepsilon g_{\alpha\varphi} + \partial_\alpha g_{\varepsilon\varphi} - \partial_\varphi g_{\alpha\varepsilon})$$

1.3.2. Consequence:

This very simple hypothesis has unexpected consequences:

(1.3.1.2.)

$$A_{\varepsilon\alpha}{}^{\gamma}.\ A^{\varepsilon}.\ g_{\gamma\beta} = \tfrac{1}{2}.\ g^{\gamma\varphi}.\ (\partial_\varepsilon g_{\alpha\varphi} + \partial_\alpha g_{\varepsilon\varphi} - \partial_\varphi g_{\alpha\varepsilon}).\ A^{\varepsilon}.\ g_{\gamma\beta} = \tfrac{1}{2}.\ \delta^{\varphi}{}_{\beta}.\ (\partial_\varepsilon g_{\alpha\varphi} + \partial_\alpha g_{\varepsilon\varphi} - \partial_\varphi g_{\alpha\varepsilon}).\ A^{\varepsilon}$$

and at the end:

(1.3.1.3.)

$$A_{\varepsilon\alpha}{}^{\gamma}.\ A^{\varepsilon}.\ g_{\gamma\beta} = \tfrac{1}{2}.\ (\partial_\varepsilon g_{\alpha\beta} + \partial_\alpha g_{\varepsilon\beta} - \partial_\beta g_{\alpha\varepsilon}).\ A^{\varepsilon}$$

For the same reasons :

(1.3.1.4.)

$$A_{\varepsilon\beta}{}^{\gamma}.\ A^{\varepsilon}.\ g_{\gamma\alpha} = \tfrac{1}{2}.\ g^{\gamma\varphi}.\ (\partial_\varepsilon g_{\beta\varphi} + \partial_\beta g_{\varepsilon\varphi} - \partial_\varphi g_{\beta\varepsilon}).\ A^{\varepsilon}.\ g_{\gamma\alpha} = \tfrac{1}{2}.\ \delta^{\varphi}{}_{\alpha}.\ (\partial_\varepsilon g_{\beta\varphi} + \partial_\beta g_{\varepsilon\varphi} - \partial_\varphi g_{\beta\varepsilon}).\ A^{\varepsilon}$$

and at the end:

(1.3.1.5.)

$$A_{\varepsilon\beta}{}^{\gamma}.\ A^{\varepsilon}.\ g_{\gamma\alpha} = \tfrac{1}{2}.\ (\partial_\varepsilon g_{\beta\alpha} + \partial_\beta g_{\varepsilon\alpha} - \partial_\alpha g_{\beta\varepsilon}).\ A^{\varepsilon}$$

These calculations, if used in (1.2.2.2) and (1.2.3.1.) above, yield:

(1.3.1.6.)

$$F = [\ldots(\partial_\alpha g_{\varepsilon\beta} - \partial_\beta g_{\varepsilon\alpha}).\ A^{\varepsilon}\ldots]$$

(1.3.1.7.)

$$[0] = G.\ \Phi - \Phi^{t}.\ G$$

These results seem to eliminate the validity of the intuitive formalism proposed at the very beginning of this theory. This is in fact an illusion. Let us make use of (1.3.1.1.) again in (1.3.1.6.). This is yielding:

(1.3.1.8.)

$$(\partial_\alpha g_{\varepsilon\beta} - \partial_\beta g_{\varepsilon\alpha}).\ A^{\varepsilon} = 2.\ [g_{\alpha\gamma}.\ A_{\varepsilon\beta}{}^{\gamma} - \partial_\varepsilon g_{\alpha\beta}].\ A^{\varepsilon}$$

Observing (1.3) and (1.10) again leads to the conclusion that:

(1.3.1.9.)

$$[\ldots(\partial_\alpha g_{\varepsilon\beta} - \partial_\beta g_{\varepsilon\alpha}).\ A^{\varepsilon}\ldots] = 2.\ G.\ \Phi - [\ldots\partial_\varepsilon g_{\alpha\beta}.\ A^{\varepsilon}\ldots]$$

Because of (1.3.1.7.) this is in fact equivalent to:

(1.3.1.10.)

$$[\ldots(\partial_\alpha g_{\varepsilon\beta} - \partial_\beta g_{\varepsilon\alpha}).\ A^{\varepsilon}\ldots] = (G.\ \Phi + \Phi^{t}.\ G) - [\ldots\partial_\varepsilon g_{\alpha\beta}.\ A^{\varepsilon}\ldots]$$

We finally get:

(1.3.1.11.)

$$F = (G.\ \Phi + \Phi^{t}.\ G) - [\ldots\partial_\varepsilon g_{\alpha\beta}.\ A^{\varepsilon}\ldots]$$
$$[0] = G.\ \Phi - \Phi^{t}.\ G$$

1.3.3. Commentary:

The Levi-Civita is the only metric compatible connection. The relation (1.3.1.11.) should be *a priori*, because of that, the only possible one.

Proposition 1.4.: Adding the Yang-Mills terms.

We can now write (0.1.) entirely.

(1.4.1.)

$$F_{\alpha\beta} = (g_{\alpha\gamma}.\ A_{\varepsilon\beta}{}^{\gamma} + A_{\varepsilon\alpha}{}^{\gamma}.\ g_{\gamma\beta} + \partial_\varepsilon g_{\alpha\beta}).\ A^{\varepsilon} + g.\ [A_\alpha, A_\beta]$$

Or, for each component (i) = 1, 2, …:

(1.4.2.)

$$F^{(i)}{}_{\alpha\beta} = (g_{\alpha\gamma}.\ A_{\varepsilon\beta}{}^{\gamma} + A_{\varepsilon\alpha}{}^{\gamma}.\ g_{\gamma\beta}).\ A^{(i)\,\varepsilon} + \partial_\varepsilon g_{\alpha\beta}.\ A^{(i)\,\varepsilon} + g.\ f^{i}{}_{jk}.\ A^{(j)}{}_{\alpha}.\ A^{(k)}{}_{\beta} \text{ for (i) = 1, 2, …}$$

Since we have made the choice for the cube A to be the Levi-Civita cube, all terms of this reduction, except the bilinear one, are depending on the metric or (or and) on the partial derivates of the metric. Note attentively that we didn't have made any choice, until now, concerning the number of the components of gauge fields. And remark that if the metric does not change (is invariant) then we stay with bilinear terms only.

2.: The gauge fields and the geometry.

Lemma 2.1. Introducing the matrix language.

Let us now consider the two last terms of the RHT of 1.4.2., i.e.:

(2.1.1.)

$$\partial_\varepsilon g_{\alpha\beta}.\ A^{(i)\,\varepsilon} + g.\ f^{(i)}{}_{jk}.\ A^{(j)}{}_{\alpha},.\ A^{(k)}{}_{\beta} \text{ for } i = 1, 2, \ldots$$

We can write them:

(2.1.2.)

$$\partial_\varepsilon g_{\alpha\beta}.\ A^{(i)\,\varepsilon} + g.\ f^{i}{}_{jk}.\ g_{\alpha\delta}.\ A^{(j)\delta}.\ g_{\beta\varepsilon}.\ A^{(k)\varepsilon}$$

Until now, we didn't say any word on the property of the multiplicative operation defined on K to be commutative or not. For the seek of simplicity we shall do as if K would be commutative and we shall come back later on this very important topic.

(2.1.3.)

$$\partial_\varepsilon g_{\alpha\beta}.\ A^{(i)\,\varepsilon} + g.\ f^{i}{}_{jk}.\ g_{\alpha\delta}.\ g_{\beta\varepsilon}.\ A^{(j)\delta}.\ A^{(k)\varepsilon}$$

We permute the structure constant involved in the above relation with components of the metric tensor:

(2.1.4.)

$$\partial_\varepsilon g_{\alpha\beta}.\ A^{(i)\,\varepsilon} + g.\ g_{\alpha\delta}.\ g_{\beta\varepsilon}.\ f^{i}{}_{jk}.\ A^{(j)\delta}.\ A^{(k)\varepsilon}$$

and we recognize the possibility to introduce another extended product on the basis where the gauge field extends. On this basis, the i-th component is:

(2.1.5.)

$$\partial_\varepsilon g_{\alpha\beta}.\ A^{(i)\,\varepsilon} + g.\ g_{\alpha\delta}.\ g_{\beta\varepsilon}.\ \triangle_{(\blacktriangledown f)}(A^{\delta}, A^{\varepsilon})^{(i)}$$

Definition 2.1.1.

The partial derivation along the variable x^λ of any element $[M] = [m_{\alpha\beta}(\ldots, x^\lambda, \ldots)]$ of M(4 x 4, K) is an endomorphism, i.e. an application ∂_λ : M(4 x 4, K) $\rightarrow$ M(4 x 4, K) | [M] $\rightarrow \partial_\lambda$ [M] = $[\partial_\lambda m_{\alpha\beta}]$.

Definition 2.1.2.

The product by a scalar of K is usually a left acting external operation on M(4 x 4, K) defined by: ".": K x M(4 x 4, K) $\rightarrow$ M(4 x 4, K) | $\forall$ k $\in$ K, k. $[M = m_{\alpha\beta}] = [k.\ m_{\alpha\beta}]$.

The matrix representation of a metric tensor within a conformal approach of the generalized theory of the relativity is symmetric and owns real components ($G = G^t$). The double reference, once to the canonical basis Ω of (E_4, K) and once to the basis of the Lie algebra under consideration is making the comprehension of these components difficult. So, taking definitions 2.1.1. and 2.1.2. into account: what we have above for the (i)-th component in the basis of the Lie algebra can also be understood as being the component in position (α, β) of the following matrix:

(2.1.6.)

$$+ A^{(i)\,\varepsilon}.\ \partial_\varepsilon G + g.\ \{G.\ T_2(\triangle_{(\blacktriangledown f)})(\mathbf{A}, \mathbf{A}).\ G\}^{(i)}$$

The really interesting result of this paragraph is the possibility to discuss about the evolution of the gauge fields in a concise way via each of the matrices:

(2.1.7.)

$$+ A^{\varepsilon}.\ \partial_\varepsilon G + g.\ \{G.\ T_2(\triangle_{(\blacktriangledown f)})(\mathbf{A}, \mathbf{A}).\ G\}$$

Remark 2.1.1.

Note that these terms are in fact corresponding to the reduction of the Faraday Maxwell tensor in inertial frames. Indeed, since we have made the choice for the cube A to be the Levi-Civita cube, and since the components of this cube vanish in any inertial frame, the $(g_{\alpha\gamma}.\ A_{\varepsilon\beta}{}^{\gamma} + A_{\varepsilon\alpha}{}^{\gamma}.\ g_{\gamma\beta}).\ A^{(i)\,\varepsilon}$ components vanish in such frames and we effectively stay with the terms 2.1.1. above.

$$F^{(i)}{}_{\alpha\beta} = \underbrace{(g_{\alpha\gamma}.\ A_{\varepsilon\beta}{}^{\gamma} + A_{\varepsilon\alpha}{}^{\gamma}.\ g_{\gamma\beta}).\ A^{(i)\,\varepsilon}}_{\text{in non inertial frames}} + \underbrace{\partial_\varepsilon g_{\alpha\beta}.\ A^{(i)\varepsilon} + g.\ f^{(i)}{}_{jk}.\ A^{(j)}{}_{\alpha}.\ A^{(k)}{}_{\beta}}_{\text{in inertial frames}}$$

2.2. Links with the Ricci flow:

2.2.1. A link with Einstein's metrics.

We must remark at this place the generality allowed for the representation of the metric tensor. This gives us the possibility to incorporate some thoughts concerning the Ricci flow into this work. A Riemannian metric is Einstein if, *per definition*:

$$R_{\alpha\beta} = \lambda.\ g_{\alpha\beta}$$

for some constant λ [02; page 174]. Note that the existence of a Einstein metric has an important consequence concerning the stress energy tensor of the generalized theory of the relativity [05; Teil V; page 116]; namely the latter becomes proportional to the metric tensor:

$$R_{\alpha\beta} - ½.\ R.\ g_{\alpha\beta} = -\ (8\pi\xi/c^4).\ T_{\alpha\beta}$$
$$\downarrow$$
$$(\lambda - ½.\ R).\ g_{\alpha\beta} = -\ (8\pi\xi/c^4).\ T_{\alpha\beta}$$

Per definition, a Ricci flow *of the type* initially defined by Hamilton on three manifolds [02; page 167; introduction] exists also in (E_4, K) if for any Greek subscripts:

$$\partial_0 g_{\alpha\beta} = -\ 2.\ R_{\alpha\beta}$$

Thus a Ricci flow in (E_4, K) with a Einstein metric would be characterized by:

$$\partial_0 g_{\alpha\beta} = -\ 2.\ \lambda.\ g_{\alpha\beta}$$

With the notation that we have introduced above in § 2.1., this state corresponds to:

$$\partial_0 G = -\ 2.\ \lambda.\ G$$

Let us now suppose, *for an illustration only*, that:

$$T_2(\triangle_{(\blacktriangledown f)})(\mathbf{A}, \mathbf{A}) = A^{\varepsilon}.\ G,$$

We get for each component (i):

$$+\ A^{\varepsilon}.\ \partial_{\varepsilon}G + g.\ \{G.\ \{(A^{\varepsilon}.\ G).\ G\}\}$$
$$=$$
$$+\ A^{\varepsilon}.\ \partial_{\varepsilon}G + g.\ \{G.\ \{A^{\varepsilon}.\ (G.\ G)\}\}$$
$$=$$
$$+\ A^{\varepsilon}.\ \partial_{\varepsilon}G + g.\ \{G.\ A^{\varepsilon}\}$$
$$=$$
$$+\ A^{\varepsilon}.\ \partial_{\varepsilon}G - g.\ A^{\varepsilon}.\ G$$
$$=$$
$$+\ A^{\varepsilon}.\ \{\partial_{\varepsilon}G + g.\ G\}$$
$$=$$
$$+\ A^0.\ \{\partial_0 G + g.\ G\} + A^1.\ \partial_1 G + A^2.\ \partial_2 G + A^3.\ \partial_3 G$$

Obviously, the first matrix of this sum is null if the coupling constant g is two times the constant making the metric being a ***Einstein***'s metric; i.e., if $g = 2.\ \lambda$. And in this case we stay with:

$$+\ A^{(i)\,1}.\ \partial_1 G + A^{(i)\,2}.\ \partial_2 G + A^{(i)\,3}.\ \partial_3 G$$

Remark 2.2.1.

This being said, the formalism of 2.1.7. which is the formalism that we stay with in inertial frames for the proposed reduction of the Faraday-Maxwell tensor suggests a more sophisticated link with the Einstein's metrics (or perhaps with a Ricci flow). Indeed, making use of the fact that the product of matrices is an associative operation, we get:

$$- A^{(i)\,\varepsilon}.\ \partial_\varepsilon G + g.\ \{\{G.\ T_2(\triangle_{(\blacktriangledown f)})(\mathbf{A}, \mathbf{A})\}.\ G\}^{(i)}$$

So, if we work on the ad hoc mathematical set (must be defined), this relation can give the sensation that g. G. $T_2(\triangle_{(\blacktriangledown f)})(\mathbf{A}, \mathbf{A})$ is a kind of eigenvalue…

Remark 2.2.2.

Quite more important: the remarks 2.1.1. and 2.2.1. lead to the idea that the absence of EM field in an inertial frame corresponds to the nullity of 2.1.1.

Note that it does not imply the nullity of the gauge fields nor of the variations of the metric. Furthermore, local and instantaneous variations of the metric that could have been neutralized or organized for a while to give us the momentary impression to be in an inertial frame without EM field, could also be at the origin of an EM field as soon as these special inertial conditions vanish. This mathematical reality could be the mechanism explaining the "birth" of some EM fields or of some EM particles in a region where nothing seemed to exist at the instant before except the geometrical background.

3. The Lorentz-Einstein Law.

3.1. Introducing the Lie Theory.

3.1.1. Conditions to get Lie brackets for the non inertial part of the reduction:

Let us a priori suppose that we start in an inertial region with no EM field and that the geometry of this region changes, giving rise to an EM field essentially described by the first part of 1.4.2. This is implicitly supposing that:

(3.1.1.1.)

$$F_{\alpha\beta} = (g_{\alpha\gamma}.\ A_{\varepsilon\beta}{}^{\gamma} + A_{\varepsilon\alpha}{}^{\gamma}.\ g_{\gamma\beta}).\ A^{\varepsilon} >> \partial_\varepsilon g_{\alpha\beta}.\ A^{\varepsilon} + g.\ [A_\alpha, A_\beta]$$

We only want to know if and when this EM field can have the formalism of a Lie bracket. That is, we actually start with:

(3.1.1.2.)

$$F = (G.\ \Phi + \Phi^t.\ G)$$

and want to define the conditions to get:

(3.1.1.3.)

$$F = [G, \Phi]$$

It is not difficult to state that the equality:

(3.1.1.4.)

$$(G.\ \Phi + \Phi^t.\ G) = [G, \Phi]$$

corresponds to:

(3.1.1.5.)

$$(G.\ \Phi + \Phi^t.\ G) = (G.\ \Phi - \Phi.\ G)$$
$$\Phi^t.\ G + \Phi.\ G = 0$$
$$(\Phi^t + \Phi).\ G = 0$$
$$\Phi^+.\ G = 0$$

where Φ^+ is the symmetric part of the trivial matrix Φ.

Remark:

When this relation 3.1.1.5. holds, this symmetric part is, in someway, orthogonal to the representation of the metric tensor. Let us recall that $so(4) = K^-{}_{44} = M^-(4 \times 4, K) = \{[M] \in M(4 \times 4, K) \mid \Phi + \Phi^t = [0]_{M(4 \times 4, K)}\}$ is the Lie algebra of the 4 x 4 skew-symmetric matrices. For the "non inertial" part of the reduction proposed for the Faraday tensor to be a Lie bracket, the matrix Φ *must not necessary* be an element of so(4). This is a remark of which we shall understand the importance in the next coming paragraphs.

3.1.2. Interpretation of the brackets with the Lie Theory.

In accordance with the spirit of the § 2. where we began to scrutinize the complicated relations between the gauge fields and the geometry, it appears to be convenient to analyze the Lie brackets obtained if 3.1.1.5. holds under the binocular of the Lie theory. Indeed, such brackets can sometimes be interpreted as being the partial derivates [03; page 262]:
(3.1.2.1.)

$$[G, \Phi] = \lim (d/dt)_{t \to 0} \exp(t. G). \Phi. \exp(-t. G)$$

or:
(3.1.2.2.)

$$- [G, \Phi] = [\Phi, G] = \lim (d/dt)_{t \to 0} \exp(t. \Phi). G. \exp(-t. \Phi)$$

if Φ and G belong to the same Lie algebra.

The latter must be defined since we know that it is not necessarily the so(4) Lie algebra (see remark above) and it cannot be the so(4) Lie algebra because G must be an element of $K^{+}_{44} = M^{+}(4 \times 4, K) = \{[M] \in M(4 \times 4, K) \mid \Phi - \Phi^t = [0]_{M(4 \times 4, K)} \}$ in a conformal approach of the relativity. And it reports on the possible variations of the metric or of the reduced representation of the Faraday-Maxwell tensor.

3.2. How can we get the Lorentz Einstein Law?

The purpose of this section is now to prove that the proposed reduction:
(3.2.1)

$$F = (G. \Phi + \Phi^t. G) - [\ldots\partial_\varepsilon g_{\alpha\beta}. A^\varepsilon\ldots] + g. [\ldots [A_\alpha, A_\beta] \ldots]$$

(3.2.2)

$$[0] = G. \Phi - \Phi^t. G$$

can be interpreted as a representation of the Lorentz-Einstein Law. For this purpose we only need to demonstrate that i) the non inertial part of the reduction of the tensor is contributing to a parallel transport and ii) that the 2.1.1. part corresponds to a proper local acceleration.

3.2.1. The idea:

One consequence of the previous section is that Φ can be seen as the representation of an infinitesimal rotation when Φ^+ vanishes because in this case Φ is reduced to its anti-symmetric part and satisfies the typical relation [04; page 19]:
(3.2.1.1)

$$\Phi^t + \Phi = 0$$

The diagonal matrix η with signature (+ , -, -, -) can be interpreted as a spinor [07].

Thus, following the spirit developed at the end of the work of E. Cartan [04], we propose to make use of the relation [04; page 147; section 174; (9)] and to write:
(3.2.1.2)

$$½. \psi. \eta = d. \eta – D. \eta$$

for *some* skew-symmetric matrix ψ. The matrices d and D are representing the "usual" and the total derivative of.

3.2.2. An instructive investigation:

The purpose of this section is to look for a part of the representation 3.2.1. of the Faraday-Maxwell tensor that could be reasonably understood as being the representation of a parallel transport. Let us calculate the following expression:
(3.2.2.1.)

$$(d. \eta – D. \eta) - (d. \eta – D. \eta)^t = [(d – D). \eta] – [(d – D). \eta]^t = [½. \psi. \eta] – [½. \psi. \eta]^t = ½. \{\psi. \eta - \eta. \psi^t\}$$

and discover if it effectively owns a formalism compatible with the Faraday-Maxwell's EM field tensor; i.e., that it is skew-symmetric; or not. We know that it does not correspond to the relation ½. {ψ. η - η. ψ} for which we want to and we can apply the idea above. But the investigation below will bring us a lot of interesting informations.

In the work of E. Cartan, d and D automatically own the following formalisms [04; page 135; section 157] and [04; page 148; section 173; (11)]:
(3.2.2.2.)

$$\begin{bmatrix} 0 & 0 & z & r^- \\ 0 & 0 & r^+ & -\bar{z} \\ -\bar{z} & r^- & 0 & 0 \\ -r^+ & -z & 0 & 0 \end{bmatrix} = \mathrm{d}$$

(3.2.2.3.)

$$\begin{bmatrix} 0 & 0 & Z & R^- \\ 0 & 0 & R^+ & -\bar{Z} \\ -\bar{Z} & R^- & 0 & 0 \\ -R^+ & -Z & 0 & 0 \end{bmatrix} = \mathrm{D}$$

where z, Z are complex numbers, z bar and Z bar the respective conjugates; where r^+ , R^+, r^- and R^- are real number with:
(3.2.2.4.)

$$Z = D_1 + i.\ D_2$$
$$z = d_1 + i.\ d_2$$
$$R^- = D_3 – (1/c).\ D_4$$
$$r^- = d_3 – (1/c).\ d_4$$
$$R^+ = D_3 + (1/c).\ D_4$$
$$r^+ = d_3 + (1/c).\ D_4$$

Let us calculate (d. η – D. η) separately:
(3.2.2.5.)

$$\begin{bmatrix} 0 & 0 & z-Z & r^- - R^- \\ 0 & 0 & r^+ - R^+ & \bar{Z}-\bar{z} \\ \bar{Z}-\bar{z} & r^- - R^- & 0 & 0 \\ r^+ - R^+ & z-Z & 0 & 0 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & Z-z & R^- - r^- \\ 0 & 0 & R^+ - r^+ & \bar{z}-\bar{Z} \\ \bar{Z}-\bar{z} & R^- - r^- & 0 & 0 \\ r^+ - R^+ & Z-z & 0 & 0 \end{bmatrix}$$

Note that:
(3.2.2.6.)

$$Z – z = (D_1 + i.\ D_2) – (d_1 + i.\ d_2) = (D_1 – d_1) + i.\ (D_2 – d_2)$$
$$(Z – z)^* = (D_1 - i.\ D_2) – (d_1 - i.\ d_2) = (D_1 – d_1) - i.\ (D_2 – d_2)$$
$$(R^+ – r^+) = (D_3 + (1/c).\ D_4) – (d_3 + (1/c).\ d_4) = (D_3 - d_3) + (1/c).\ (D_4 – d_4)$$
$$(R^- – r^-) = (D_3 - (1/c).\ D_4) – (d_3 - (1/c).\ d_4) = (D_3 - d_3) - (1/c).\ (D_4 – d_4)$$

Let us remember that [06; page 73; (23,5)]:
(3.2.2.7.)

$$\begin{bmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & -H^3 & H^2 \\ -E^2 & H^3 & 0 & -H^1 \\ -E^3 & -H^2 & H^1 & 0 \end{bmatrix} = [F]$$

Remark:
An identification with the Faraday-Maxwell’s EM field tensor:

$$½.\ \psi.\ \eta = d.\ \eta – D.\ \eta = F$$

would imply:
(3.2.2.8.)

$$E^1 = 0$$

(3.2.2.9.)

$$E^2 = (D_1 - d_1) - i.\ (D_2 - d_2) = -\ [(D_1 - d_1) + i.\ (D_2 - d_2)] \rightarrow (D_1 - d_1) = 0 \rightarrow E^2 = -i.\ (D_2 - d_2)$$

(3.2.2.10.)

$$E^3 = -[(D_3 - d_3) + (1/c).\ (D_4 - d_4)] = -\ (D_3 - d_3) + (1/c).\ (D_4 - d_4) \rightarrow (D_4 - d_4) = 0 \rightarrow E^3 = -\ (D_3 - d_3)$$

(3.2.2.11.)

$$H^1 = 0$$

(3.2.2.12.)

$$H^2 = -\ (D_1 - d_1) + i.\ (D_2 - d_2) = -\ [(D_1 - d_1) + i.\ (D_2 - d_2)] \rightarrow (D_2 - d_2) = 0 \rightarrow H^2 = -\ (D_1 - d_1) = 0$$

(3.2.2.13.)

$$H^3 = (D_3 - d_3) - (1/c).\ (D_4 - d_4) = -\ [(D_3 - d_3) + (1/c).\ (D_4 - d_4)] \rightarrow (D_3 - d_3) = 0 \rightarrow H^3 = -\ (1/c).\ (D_4 - d_4) = 0$$

This identification does not make sense because it is yielding at the end:
(3.2.2.14.)

$$\mathbf{E = H = 0}$$

Now let us come back to the purpose of this section and calculate:
(3.2.2.15.)

$$(½.\ \psi.\ \eta)^t = (d.\ \eta - D.\ \eta)^t = \eta.\ \{d - D\}^t$$

This is in extenso:
(3.2.2.16.)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} . \begin{bmatrix} 0 & 0 & \overline{Z}-\overline{z} & r^+-R^+ \\ 0 & 0 & r^- - R^- & z-Z \\ z-Z & r^+-R^+ & 0 & 0 \\ r^- - R^- & \overline{Z}-\overline{z} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \overline{Z}-\overline{z} & r^+-R^+ \\ 0 & 0 & R^- - r^- & Z-z \\ Z-z & R^+-r^+ & 0 & 0 \\ R^- - r^- & \overline{z}-\overline{Z} & 0 & 0 \end{bmatrix}$$

And we must absolutely calculate:
(3.2.2.17.)

$$(d.\ \eta - D.\ \eta) - (d.\ \eta - D.\ \eta)^t$$

$$=$$

$$\begin{bmatrix} 0 & 0 & Z-z & R^- - r^- \\ 0 & 0 & R^+ - r^+ & \overline{z}-\overline{Z} \\ \overline{Z}-\overline{z} & R^- - r^- & 0 & 0 \\ r^+ - R^+ & Z-z & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & \overline{Z}-\overline{z} & r^+-R^+ \\ 0 & 0 & R^- - r^- & Z-z \\ Z-z & R^+-r^+ & 0 & 0 \\ R^- - r^- & \overline{z}-\overline{Z} & 0 & 0 \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} 0 & 0 & (Z-\overline{Z})-(z-\overline{z}) & (R^+ + R^-)-(r^+ + r^-) \\ 0 & 0 & (R^+ - R^-)-(r^+ - r^-) & -(Z+\overline{Z})+(z+\overline{z}) \\ -(Z-\overline{Z})+(z-\overline{z}) & -(R^+ - R^-)+(r^+ - r^-) & 0 & 0 \\ -(R^+ + R^-)+(r^+ + r^-) & (Z+\overline{Z})-(z+\overline{z}) & 0 & 0 \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} 0 & 0 & (Z+\overline{z})-(\overline{Z}+z) & (R^{+}+R^{-})-(r^{+}+r^{-}) \\ 0 & 0 & (R^{+}-r^{+})-(R^{-}-r^{-}) & -(Z-\overline{z})-(\overline{Z}-z) \\ -(Z+\overline{z})+(\overline{Z}+z) & -(R^{+}-r^{+})+(R^{-}-r^{-}) & 0 & 0 \\ -(R^{+}+R^{-})+(r^{+}+r^{-}) & (Z-\overline{z})+(\overline{Z}-z) & 0 & 0 \end{bmatrix}$$

Note that:
(3.2.2.18.)

$$Z + z^* = (D_1 + i.\ D_2) + (d_1 - i.\ d_2) = (D_1 + d_1) + i.\ (D_2 - d_2)$$
$$Z - z^* = (D_1 + i.\ D_2) - (d_1 - i.\ d_2) = (D_1 - d_1) + i.\ (D_2 + d_2)$$
$$Z^* - z = (D_1 - i.\ D_2) - (d_1 + i.\ d_2) = (D_1 - d_1) - i.\ (D_2 + d_2)$$
$$Z^* + z = (D_1 - i.\ D_2) + (d_1 + i.\ d_2) = (D_1 + d_1) - i.\ (D_2 - d_2)$$
$$(R^+ - r^+) = (D_3 + (1/c).\ D_4) - (d_3 + (1/c).\ d_4) = (D_3 - d_3) + (1/c).\ (D_4 - d_4)$$
$$(R^- - r^-) = (D_3 - (1/c).\ D_4) - (d_3 - (1/c).\ d_4) = (D_3 - d_3) - (1/c).\ (D_4 - d_4)$$

Consequently:
(3.2.2.19.)

$$(Z - z^*) + (Z^* - z) = 2.\ (D_1 - d_1)$$
$$(Z + z^*) - (Z^* + z) = 2i.\ (D_2 - d_2)$$
$$(R^+ - r^+) + (R^- - r^-) = 2.\ (D_3 - d_3)$$
$$(R^+ - r^+) - (R^- - r^-) = (2/c).\ (D_4 - d_4)$$

And conclude that:
(3.2.2.20.)

$$(d.\ \eta - D.\ \eta) - (d.\ \eta - D.\ \eta)^t$$
$$=$$
$$\begin{bmatrix} 0 & 0 & 2i.\ (D_2-d_2) & 2.\ (D_3-d_3) \\ 0 & 0 & (2/c).\ (D_4-d_4) & -2i.\ (D_1-d_1) \\ -2i.\ (D_2-d_2) & -(2/c).\ (D_4-d_4) & 0 & 0 \\ -2.\ (D_3-d_3) & 2i.\ (D_1-d_1) & 0 & 0 \end{bmatrix}$$

The ½. {ψ. η - η. ψ^t} matrix owns *a priori* (see the analysis below) a quite more interesting formalism even if all components are not real. A comparison with the formalism of 3.2.2.7. can be proposed in a coherent manner if we accept imaginary components for the EM field. The formulation of EM laws with complex numbers is something usual in modern physics [06]. If a rationalistic thinker cannot accept this fact, it is always possible to reduce the validity of 3.2.2.20. to cases where $(D_2 - d_2) = (D_1 - d_1) = 0$. This eliminates EM fields with imaginary components and limit the discussion to an identification with EM fields owning only one electrical component along the Oy axis and only one magnetic component along the Oz axis. These special situations correspond exactly to some plane waves in vacuum. Otherwise, even a rationalistic thinker should not reject the imaginary components of the EM fields because they are reporting on the polarizations.

3.2.3. Analysis:

If the relation 3.2.1.2. holds, the calculation made above corresponds to ½. {ψ. η - η. ψ^t}. Following the spirit of the work of E. Cartan, the relation 3.2.1.2. only makes sense if ψ is the representation of an infinitesimal rotation, i.e. is a skew-symmetric matrix, that is satisfies 3.2.1.1. We thus have made the demonstration that {ψ. η + η. ψ} is a skew symmetric matrix if ψ is itself one.

Remark:
But as a matter of fact,
(3.2.3.1)

$$\forall\ \psi \in K^{-}_{44},\ \lim_{G \to \eta} \{G.\ \psi - \psi^t.\ G\} = \{\eta.\ \psi + \psi.\ \eta\}$$

The conclusion is that if the reduction 3.2.1. of the Faraday-Maxwell tensor makes sense and if the interpretation of this reduction via the work of E. Cartan can be done with the proposition 3.2.1.2, the circumstance:

(3.2.3.2)

$$\psi = \Phi \in so(4)$$

corresponds to the necessary absence of parallel transport induced by our special interpretation when the reduction occurs in a Minkowskian metric supposed to be compatible with the local connection. Indeed the compatibility between the metric and the connection has a price, namely the relation 3.2.2. above: $G.\ \Phi - \Phi^t.\ G = [0]_{M(4 \times 4, K)}$.

Stricto sensu, the work of E. Cartan leads us to test in the same way as above if the matrix ½. {ψ. η - η. ψ} corresponds to a parallel transport or not.

3.2.4. A new try:

This section only repeats the calculations made in § 3.2.2. above, but for the matrix ψ. η - η. ψ. Since we know what the matrix ψ. η is, we have to calculate η. ψ, or equivalently η. (d - D) if we continue to interpret η. ψ as a small parallel transport induced by the infinitesimal rotation ψ and the spinor η. This is:

(3.2.4.1)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} . \begin{bmatrix} 0 & 0 & z-Z & r^- - R^- \\ 0 & 0 & r^+ - R^+ & \overline{Z} - \overline{z} \\ \overline{Z} - \overline{z} & r^- - R^- & 0 & 0 \\ r^+ - R^+ & z - Z & 0 & 0 \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} 0 & 0 & z-Z & r^- - R^- \\ 0 & 0 & -(r^+ - R^+) & -(\overline{Z} - \overline{z}) \\ -(\overline{Z} - \overline{z}) & -(r^- - R^-) & 0 & 0 \\ -(r^+ - R^+) & -(z - Z\ ) & 0 & 0 \end{bmatrix}$$

From this we deduce:

(3.2.4.2)

$$\psi.\ \eta - \eta.\ \psi$$

$$=$$

$$\begin{bmatrix} 0 & 0 & Z-z & R^- - r^- \\ 0 & 0 & R^+ - r^+ & \overline{z} - \overline{Z} \\ \overline{Z} - \overline{z} & R^- - r^- & 0 & 0 \\ r^+ - R^+ & Z - z & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & z-Z & r^- - R^- \\ 0 & 0 & -(r^+ - R^+) & -(\overline{Z} - \overline{z}) \\ -(\overline{Z} - \overline{z}) & -(r^- - R^-) & 0 & 0 \\ -(r^+ - R^+) & -(z - Z\ ) & 0 & 0 \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2.\ (R^+ - r^+) & 2.\ (\overline{z} - \overline{Z}) \\ 2.\ (\overline{Z} - \overline{z}) & 2.\ (R^- - r^-) & 0 & 0 \\ 2.\ (r^+ - R^+) & 2.\ (Z - z) & 0 & 0 \end{bmatrix}$$

Let us remember the results 3.2.2.18. and continue with:

(3.2.4.3)

$$=$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2.[(D_3-d_3)+(1/c).(D_4-d_4)] & -2.[(D_1-d_1)-i.(D_2-d_2)] \\ 2.[(D_1-d_1)-i.(D_2-d_2)] & 2.[(D_3-d_3)-(1/c).(D_4-d_4)] & 0 & 0 \\ -2.[(D_3-d_3)+(1/c).(D_4-d_4)] & 2.[(D_1-d_1)+i.(D_2-d_2)] & 0 & 0 \end{bmatrix}$$

Let us remark that if the differential operators D_α and d_α are *real* one, then an identification of this matrix with the matrix representing the Faraday Maxwell tensor makes only sense if:

$$(D_2 - d_2) = (D_1 - d_1) = 0$$
$$(D_3 - d_3) - (1/c).\ (D_4 - d_4) = 0$$
$$(D_3 - d_3) + (1/c).\ (D_4 - d_4) = 0$$

… the faraday Maxwell tensor vanishes…

3.2.5. Going further in the analysis.

What can we learn from this? The relation 3.2.2.17. shows that (d. η – D. η) + (d. η – D. η)^t is a symmetric matrix and can also *not* be understood as being a representation of F. We finally have discovered only one family of matrices that could give us a realistic representation of a parallel transport, namely the (d. η – D. η) - (d. η – D. η)^t matrices. These matrices correspond to ½. {ψ. η - η. ψ^t} with ψ ∈ so(4). We logically did deduce from these two relations that it was concerning the ½. {ψ. η + η. ψ} matrices for which we have also demonstrated that they were always null *if* ψ = Φ because of the compatibility between the connection and the metric. This is suggesting that the multiplication becomes, at the limit: ψ = Φ, an anti-commutative operation on a certain subset of M(4 x 4, K) x M(4 x 4, K). And once more time in this work, we meet the complicated notion of anti-commutative product.

Note that if the multiplication would be a commutative operation on M(4 x 4, K), then we would have: ψ. η = η. ψ and ½. {ψ. η + η. ψ} = ψ. η; finally we would have for the limit case ψ = Φ: Φ. η = infinitesimal rotation x spinor = zero matrix because of the compatibility between the connection and the metric. Note that such a relation is a typical one in the theory of Cartan, i.e. is characteristic of the spinoral property of η if one consider that Φ is the representation of the 4 equations associated with a isotropic 2-plane [04; page 93].

In fact, we just have seen with the relation 3.2.4.3. that ψ. η is different of η. ψ in general, *except if* we try to identify the difference with the representation of the Faraday-Maxwell tensor.

The reduction 3.2.1. proposed for the tensor F in this work does absolutely not pre-impose any condition to the matrix Φ except the relation 3.2.2. to insure the compatibility between the connection and the metric. This means that we should always write:
(3.2.5.1)

$$\Phi = \Phi^+ + \Phi^-$$

and make use of this relation in the reduction of F to find the part of it that can be at the origin of a parallel transport.

$$F = (G.\ \Phi + \Phi^t.\ G) = G.\ (\Phi^+ + \Phi^-) + (\Phi^+ + \Phi^-)^t.\ G + \ldots$$
$$0 = G.\ \Phi - \Phi^t.\ G = G.\ (\Phi^+ + \Phi^-) - (\Phi^+ + \Phi^-)^t.\ G$$

Since $\Phi^+ \in M^+(4 \times 4, K)$ and $\Phi^- \in so(4)$, we get:
(3.2.5.2)

$$\Phi^{+\,t} = \Phi^+$$

(3.2.5.3)

$$\Phi^{-\,t} = -\ \Phi^-$$

(3.2.5.4)

$$F = G.\ \Phi^+ + G.\ \Phi^- + \Phi^{+\,t}.\ G + \Phi^{-t}.\ G = (G.\ \Phi^+ + \Phi^+.\ G) + (G.\ \Phi^- - \Phi^-.\ G) + \ldots$$

(3.2.5.5)

$$0 = (G.\ \Phi^+ - \Phi^+.\ G) + (G.\ \Phi^- + \Phi^-.\ G)$$

This yields very interesting new informations. First of all, we recognize that the second term of the RHT of 3.2.5.5. can be interpreted as a parallel transport matrix (see § 3.2.2) when the metric reaches the Minkowskian limit. From this fact, logically, because $\eta.\ \Phi^- + \Phi^-.\ \eta$ is also an element of so(4), we deduce:

(3.2.5.6)

$$\eta.\ \Phi^+ - \Phi^+.\ \eta = -(\eta.\ \Phi^- + \Phi^-.\ \eta) \in so(4)$$

There is another family of matrices built with the symmetric part of the matrix Φ and able to be understood as a parallel transport matrix …

Now let us consider the reduction of the tensor at the Minkowskian limit; we can always write:

(3.2.5.7)

$$F = (\eta.\ \Phi^+ + \Phi^+.\ \eta) + (\eta.\ \Phi^- - \Phi^-.\ \eta) + \ldots = (\eta.\ \Phi^+ - \Phi^+.\ \eta) + (\eta.\ \Phi^- + \Phi^-.\ \eta) + 2.\ (\Phi^+.\ \eta - \Phi^-.\ \eta) + \ldots = 2.\ (\Phi^+.\ \eta - \Phi^-.\ \eta) + \ldots$$

But the interesting thing for us is to write:

(3.2.5.8)

$$F = (\eta.\ \Phi^- + \Phi^-.\ \eta) + (\eta.\ \Phi^+ + \Phi^+.\ \eta) - 2.\ \Phi^-.\ \eta + \ldots$$

where we now know that the first term in the RHT is a parallel transport matrix.

$$F = \begin{bmatrix} 0 & 0 & 2i.\ (D_2 - d_2) & 2.\ (D_3 - d_3) \\ 0 & 0 & (2/c).\ (D_4 - d_4) & -2i.\ (D_1 - d_1) \\ -2i.\ (D_2 - d_2) & -(2/c).\ (D_4 - d_4) & 0 & 0 \\ -2.\ (D_3 - d_3) & 2i.\ (D_1 - d_1) & 0 & 0 \end{bmatrix} + (\eta.\ \Phi^+ + \Phi^+.\ \eta) - 2.\ \Phi^-.\ \eta + \ldots$$

3.3. Conclusion:

Any EM field represented by its matrix F in so(4) can be reduced into two parts: a "non inertial" part, containing itself a parallel transport component, and an inertial one.

If we can complete this work with a relation like **A** = [T]. **v**, describing the transformation between the EM potential four-vector and the local proper speed vector, this way of thinking gives us the possibility to built a formal link with the Lorentz-Einstein Law. This topic must be developed.

One can argue that such a reduction looks a little bit forced or artificial but the only fact that it exists can give us an interesting and natural tool to describe the behavior of any EM field in presence of a gravitational field. It also gives us the self contained gravitational part of any EM field, the latter being considered as an energy carrier or source.

We shall continue this investigation further in the next part, studying the implication of our way of thinking on the expression of the Lagrangian and on the expression of the action. We can still note that our theory, at this moment, introduces no contradiction with a usual way of doing in the construction of a Yang-Mills theory. For example there is no reason to believe that some invariance will not be respected because of this way of doing. It will just give us a new insight in the behavior of the nature that we propose to observe with new binoculars but not to change.

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